Multi-frequency Methods for Solving **Inverse Problems** in **TOF Imaging**

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iTOF Workshop Ein Gedi *2014*

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Acknowledgements

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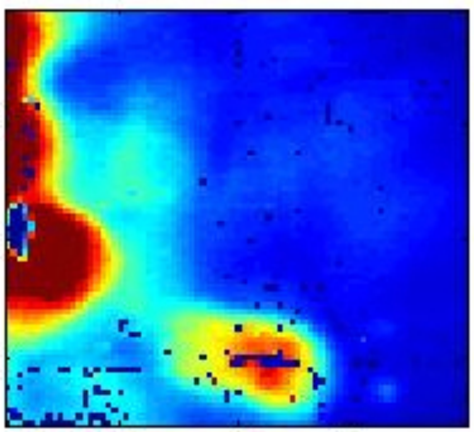
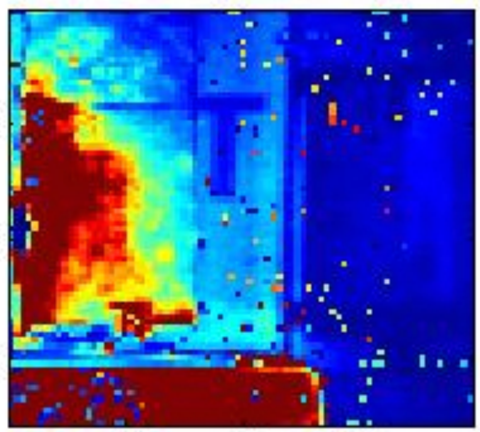
Mission Statement

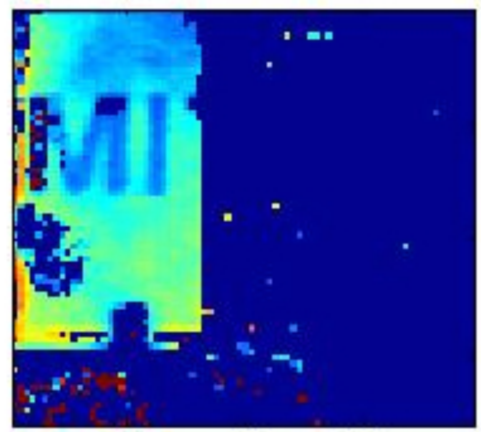
— Provide a unified perspective for inverse problems in ToF imaging.

— Develop a Toolbox that is applicable to wide variety of applications for ToF Imaging.

— Demonstrate practicability of signal processing algorithms in context of ToF Imaging.

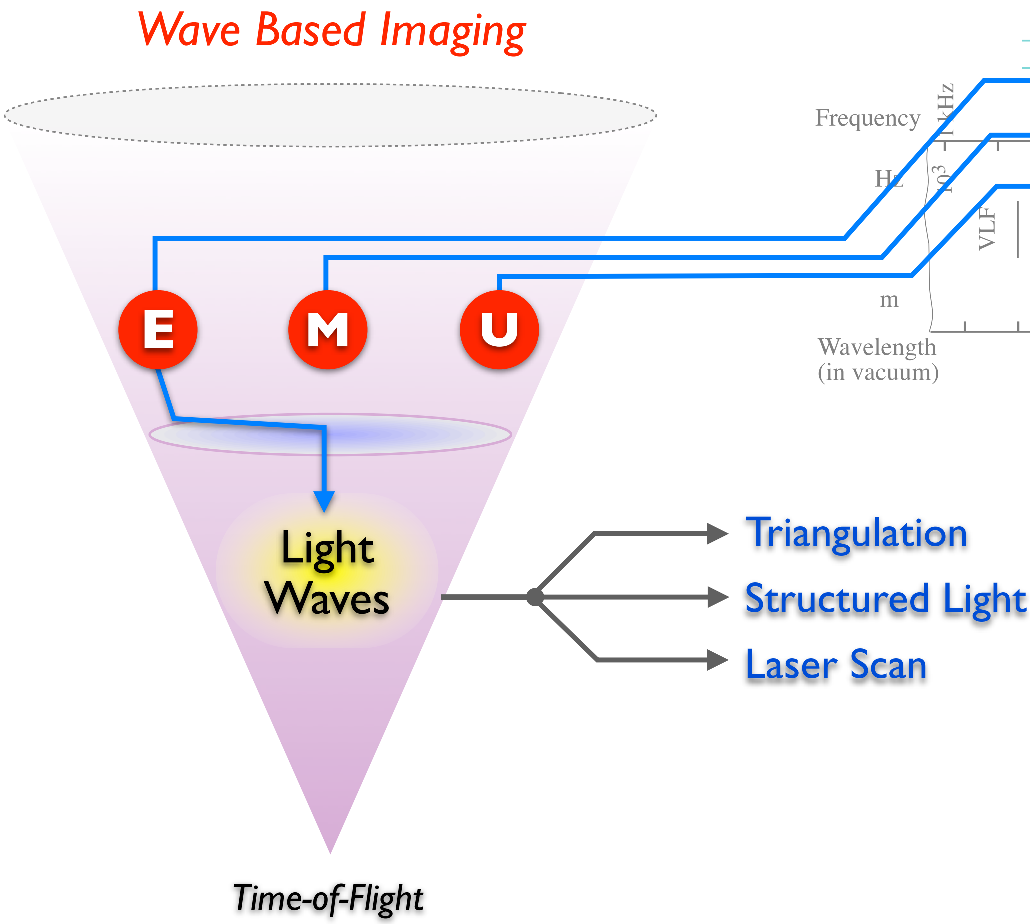


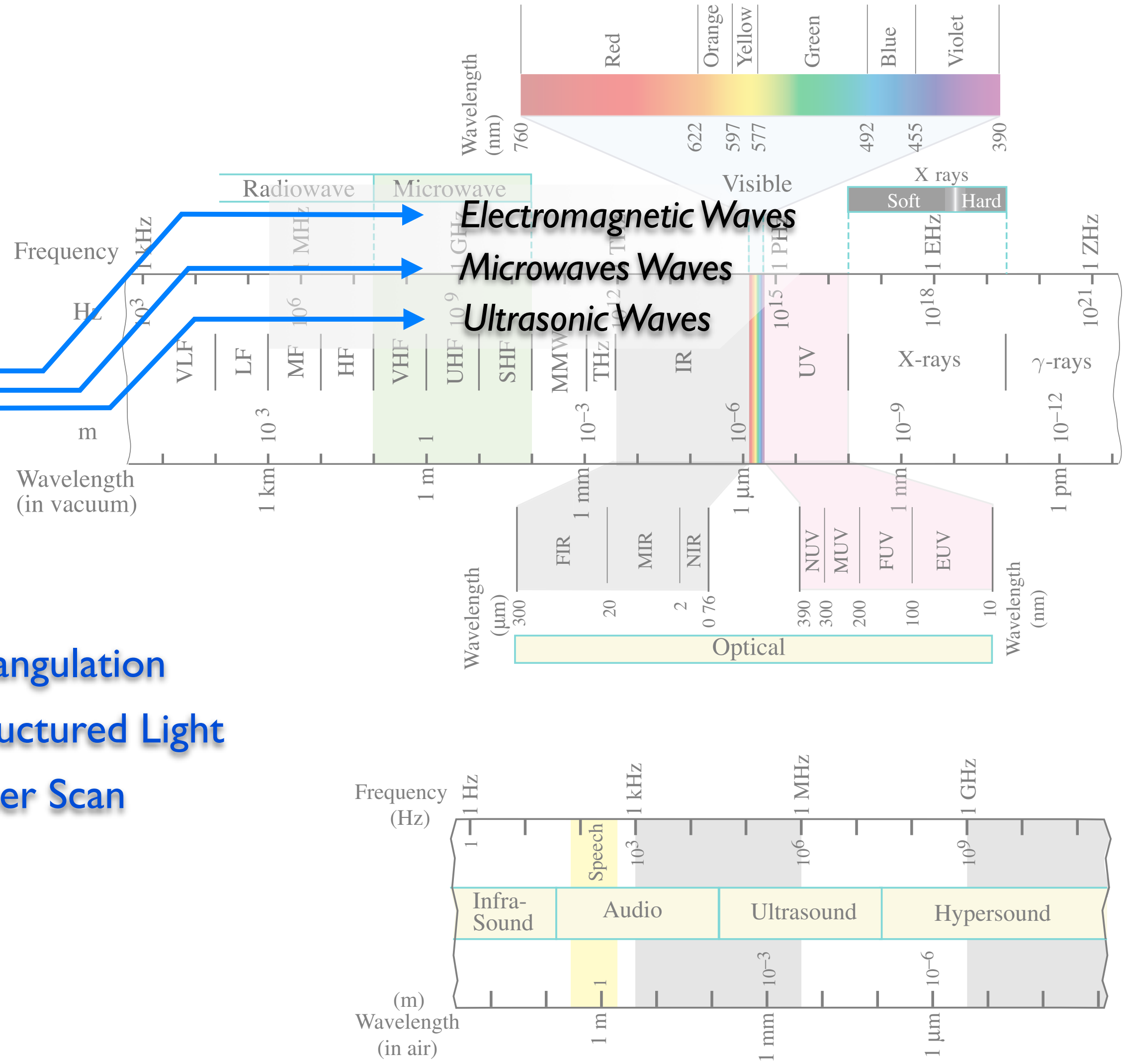




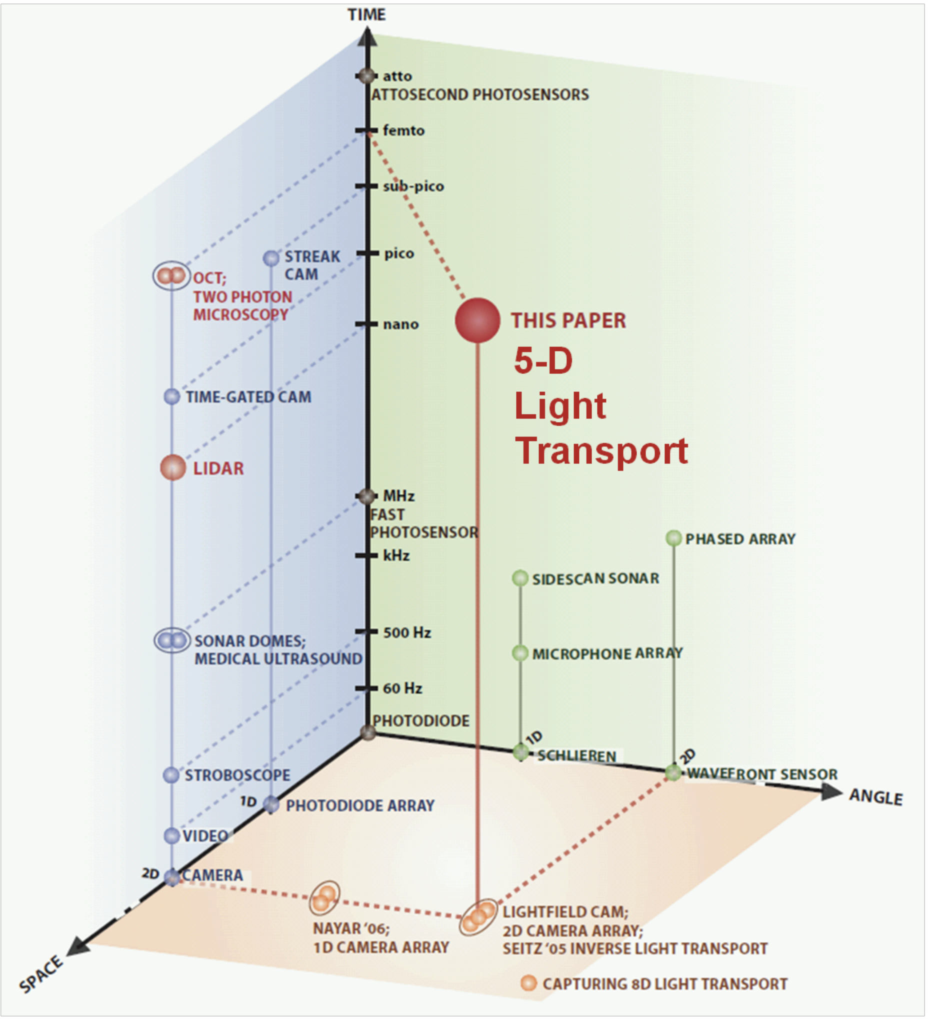
Example of Reconstruction

Depth Imaging/3D Imaging

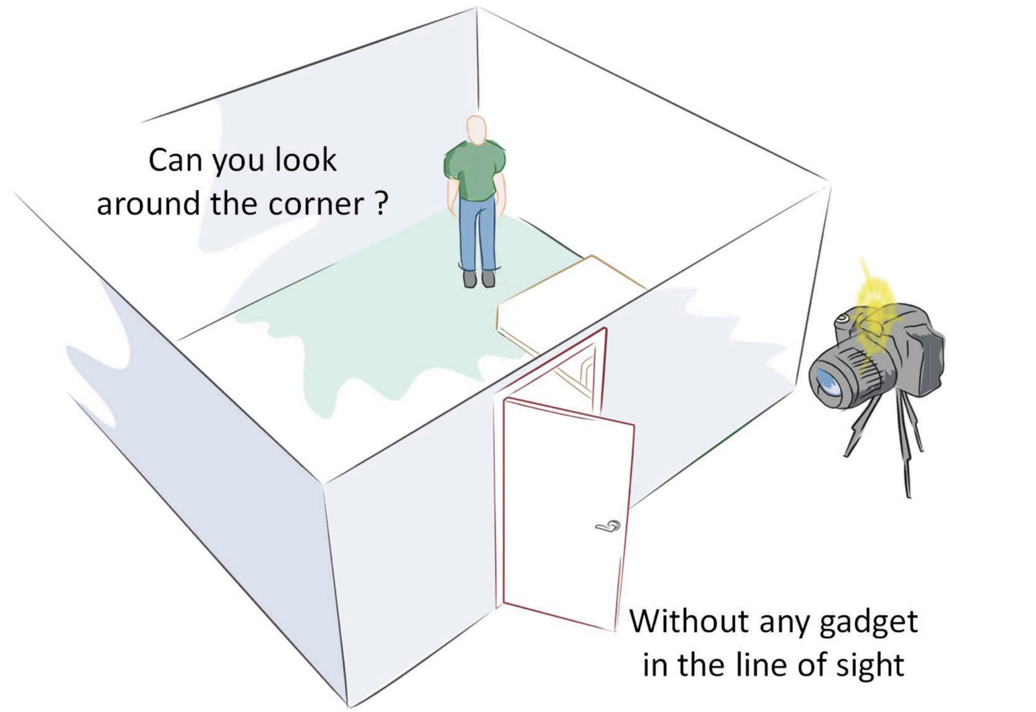




Time of Flight work at MIT



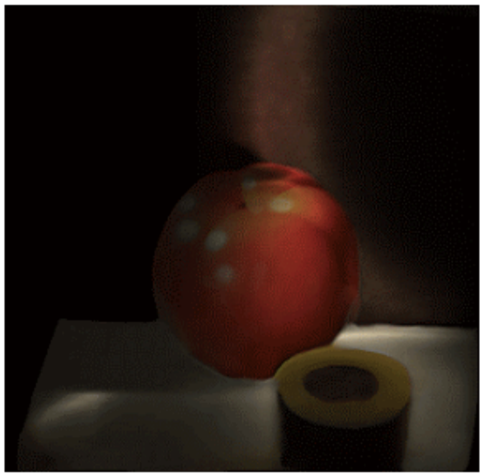
Raskar / J. Davies (2005)



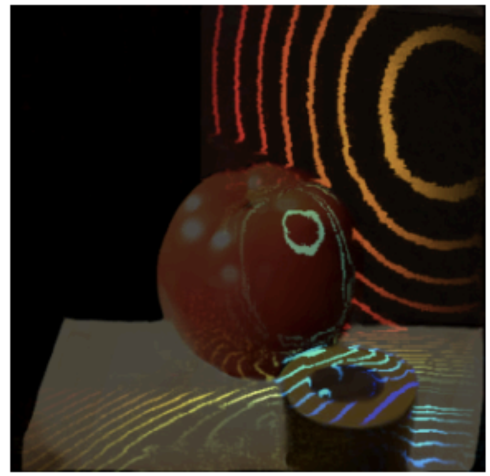
Velten et al Nature Comm.

Time of Flight work at MIT

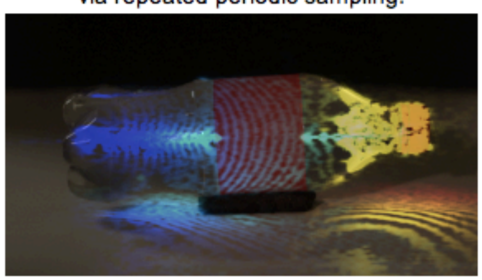
Femto-Photography: Visualizing Photons in Motion at a Trillion Frames Per Second



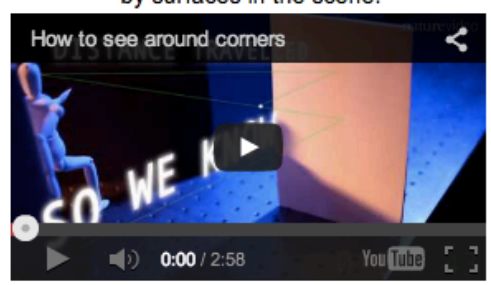
Light in Motion: Combination of modern imaging hardware and a reconstruction technique to visualize light propagation via repeated periodic sampling.



Ripples of Waves: A time-lapse visualization of the spherical fronts of advancing light reflected by surfaces in the scene.



Time-Lapse Visualization: Color coding of light with a delay of few picoseconds in each period.

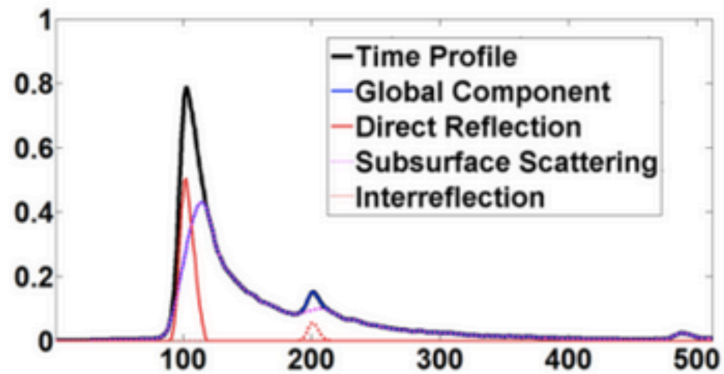


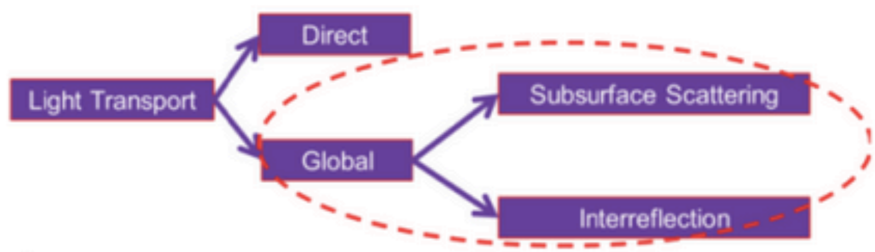
Video by Nature.com
Nature.com/news

Time of Flight work at MIT

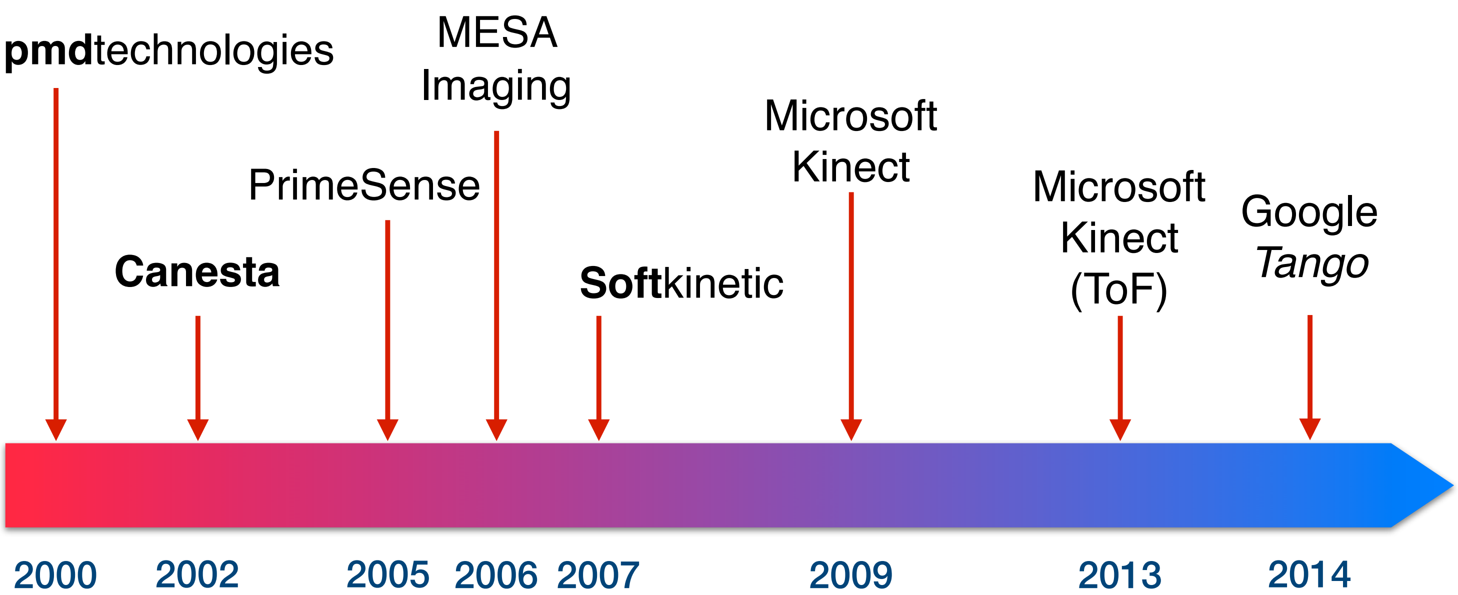
Decomposing Global Light Transport using Time of Flight Imaging

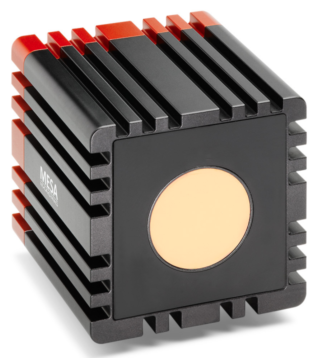
Di Wu, Matthew O'Toole, Andreas Velten, Amit Agrawal, and Ramesh Raskar.
Computer Vision and Pattern Recognition (CVPR), 2012. (Oral)



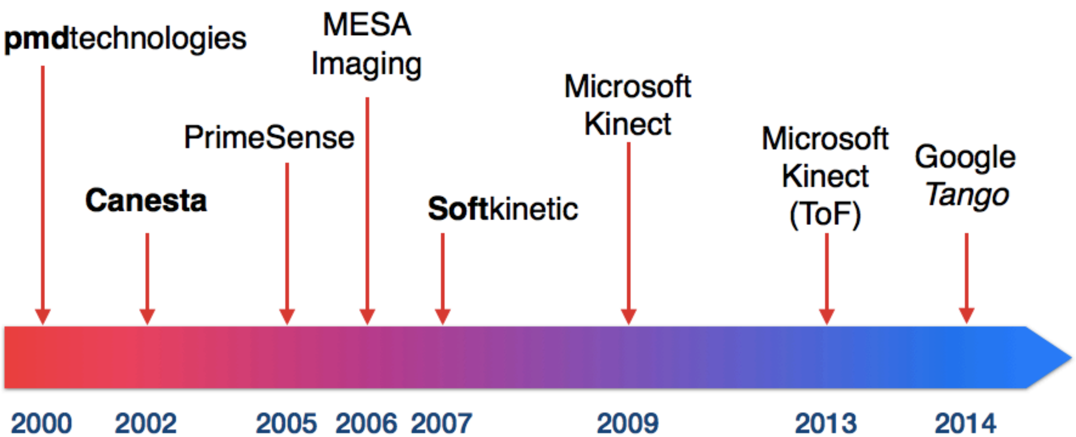


The Rise and Rise of Depth Imaging

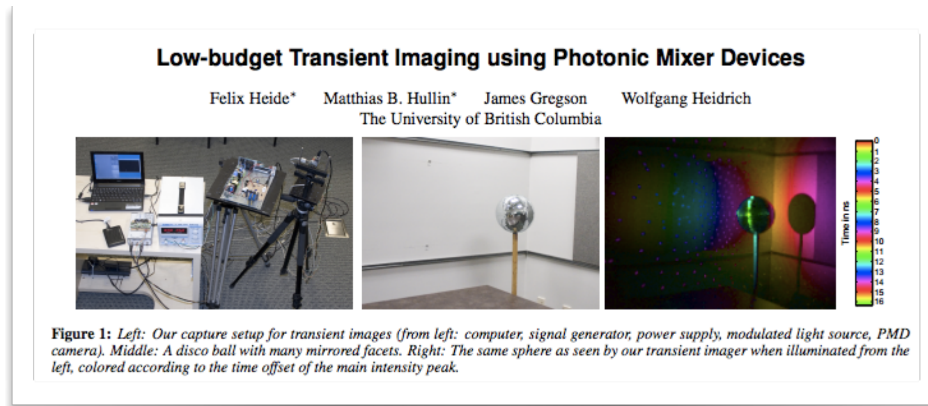




The Rise and Rise of Depth Imaging



Heide/Hullin

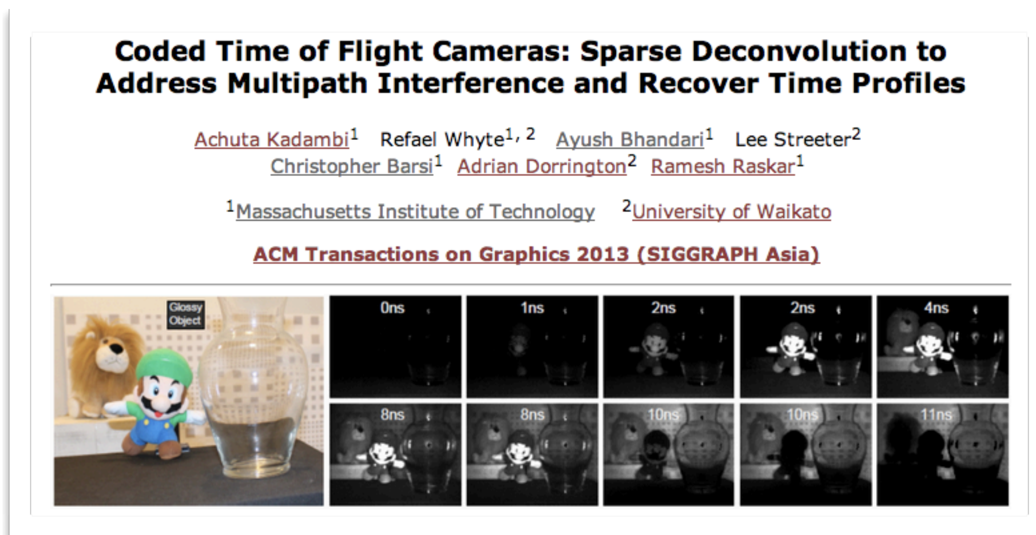


Low-budget Transient Imaging using Photonic Mixer Devices

Felix Heide* Matthias B. Hullin* James Gregson Wolfgang Heidrich
The University of British Columbia

Figure 1: *Left: Our capture setup for transient images (from left: computer, signal generator, power supply, modulated light source, PMD camera). Middle: A disco ball with many mirrored facets. Right: The same sphere as seen by our transient imager when illuminated from the left, colored according to the time offset of the main intensity peak.*

MIT Team



Coded Time of Flight Cameras: Sparse Deconvolution to Address Multipath Interference and Recover Time Profiles

Achuta Kadambi[1] Refael Whyte[1, 2] Ayush Bhandari[1] Lee Streeter[2]
Christopher Barsi[1] Adrian Dorrington[2] Ramesh Raskar[1]

[1]Massachusetts Institute of Technology [2]University of Waikato

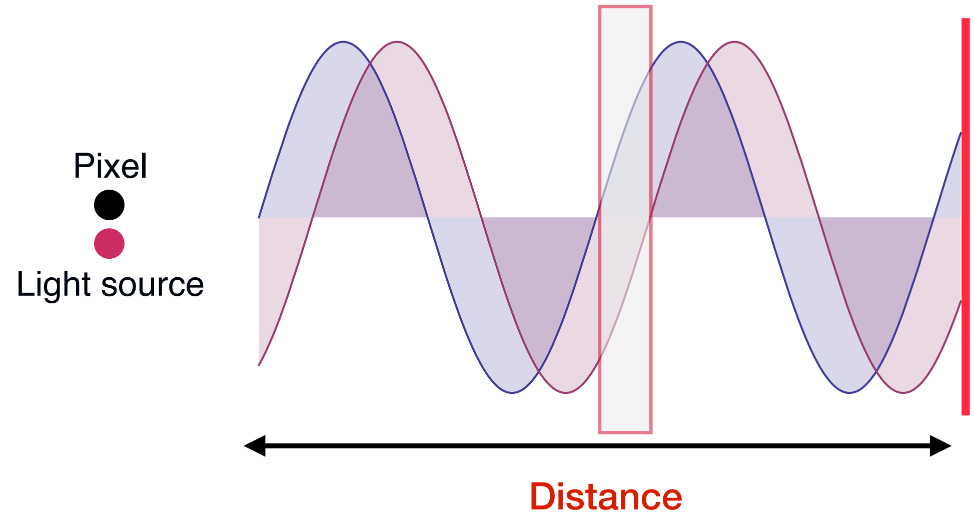
ACM Transactions on Graphics 2013 (SIGGRAPH Asia)

— Applications beyond depth imaging:

- Multi-bounce light decomposition
- Ultra-fast imaging
- Bio-imaging
- Scattering medium

Depth from Continuous Wave Imaging

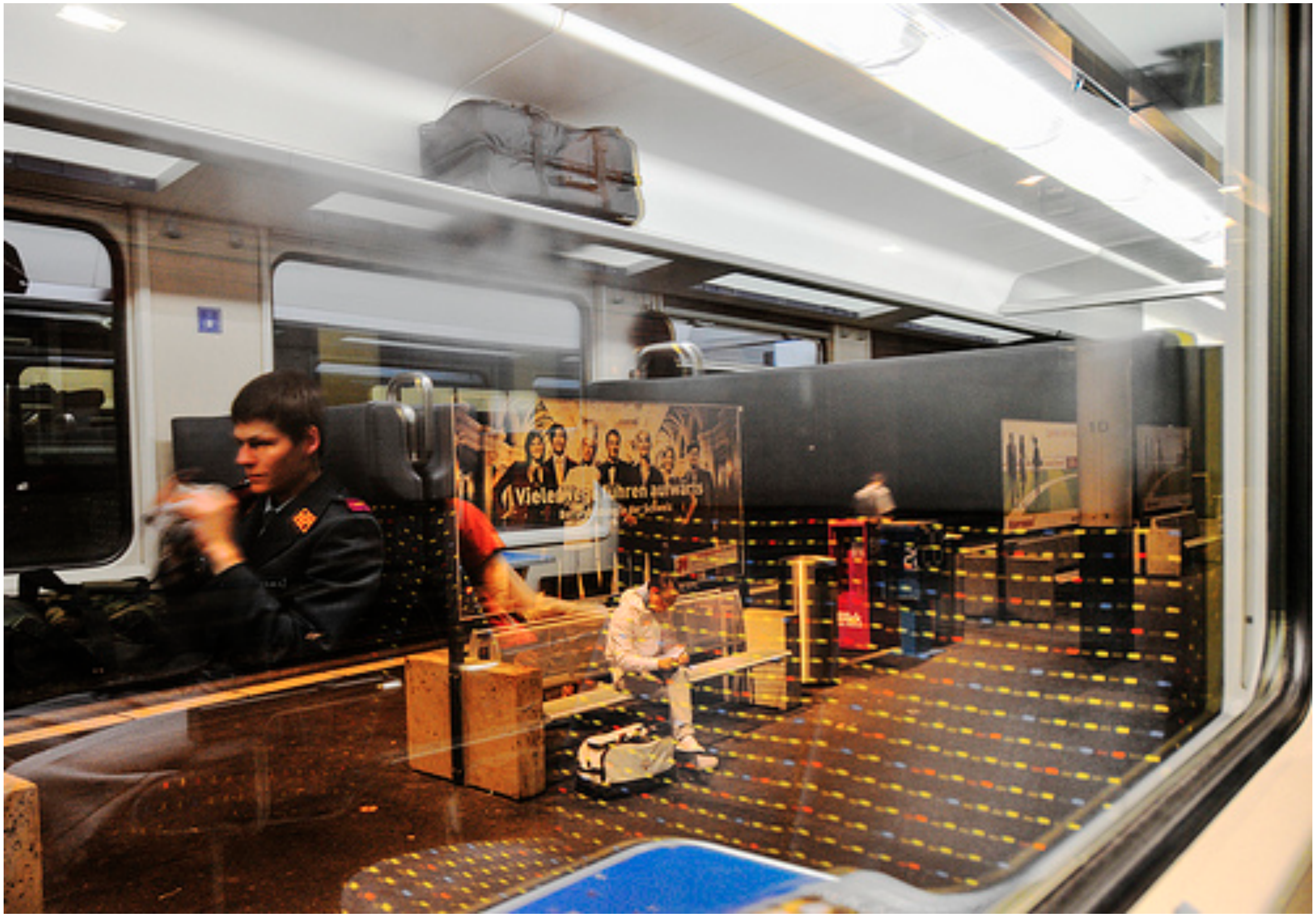
Frequency Domain



Time Domain

Example: Femto-photography

Multiple Depths / Mixed Pixels — Problem



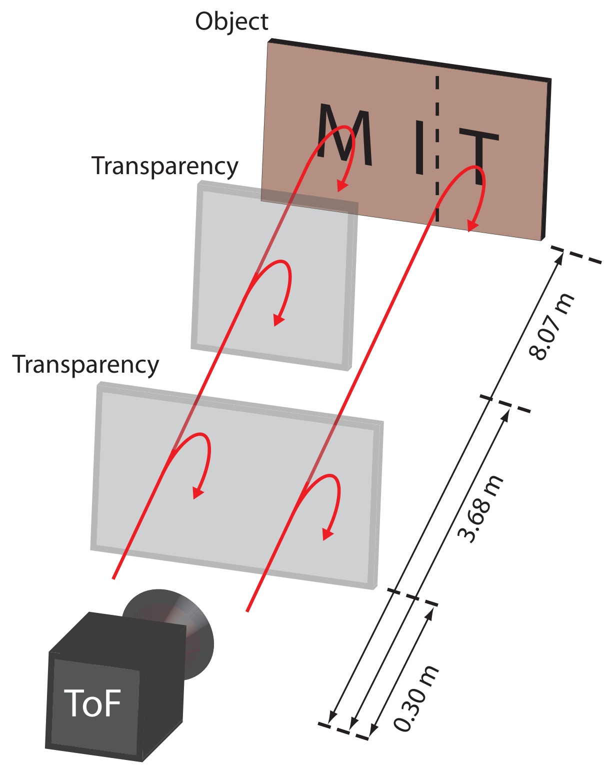
Multiple Depths / Mixed Pixels — Problem

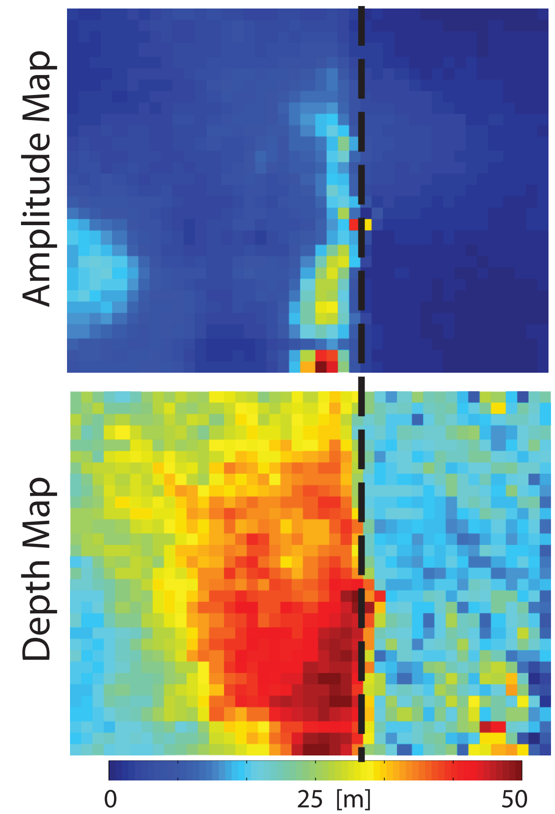
$$z_\omega^{(K)} = \tilde{\Gamma}(\omega) e^{j\tilde{\phi}(\omega)} = \sum_{k=0}^{K-1} \Gamma_k e^{j\phi_k(\omega)},$$

in $\mathbf{\Psi}$. We can also define a vector $\boldsymbol{g}' \in \mathbb{R}^{L\times 1}$, whose elements are zero except for K reflection amplitudes $\{\Gamma_k\}_{k=0}^{K-1}$, such that $\mathbf{z} = \mathbf{\Psi}\boldsymbol{g}'$. We use the ($K$-) sparsity of $\boldsymbol{g}'$ to regularize the problem:

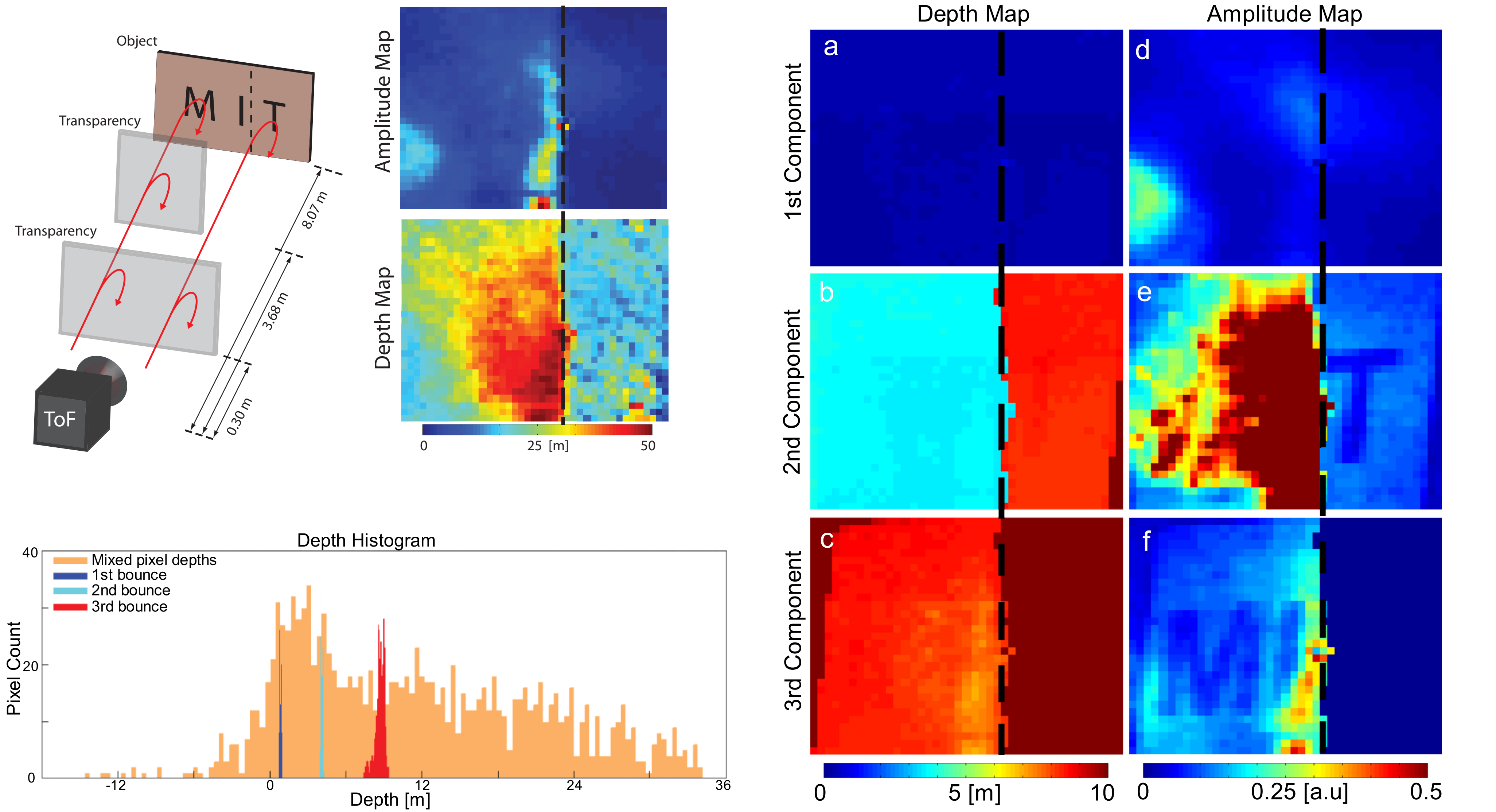
$$\underbrace{\|\mathbf{z} - \mathbf{\Psi}\boldsymbol{g}'\|^2_{\ell_2}}_{\text{Data-Fidelity}} < \varepsilon_0 \quad \text{such that} \quad \underbrace{\|\boldsymbol{g}'\|_{\ell_0}}_{\text{Sparsity}} = K, \qquad (11)$$

where the ℓ_p-norm is $\|\mathbf{x}\|^p_{\ell_p} \stackrel{\text{def}}{=} \sum_n |x_n|^p$. The limit $p \to 0$ is used to define $\|\boldsymbol{g}'\|_{\ell_0}$ as the number of nonzero elements of $\boldsymbol{g}'$. Equation (11) demands a least-squares solution to





Multiple Depths / Mixed Pixels — Solution



Details …

A. Bhandari, A. Kadambi, R. Whyte, C. Barsi, M. Feigin, A. Dorrington, and R. Raskar, "Resolving multipath interference in time-of-flight imaging via modulation frequency diversity and sparse regularization," *Optics Letters*, vol. 39, no. 7, 2014.

April 1, 2014 / Vol. 39, No. 7 / OPTICS LETTERS

Resolving multipath interference in time-of-flight imaging via modulation frequency diversity and sparse regularization

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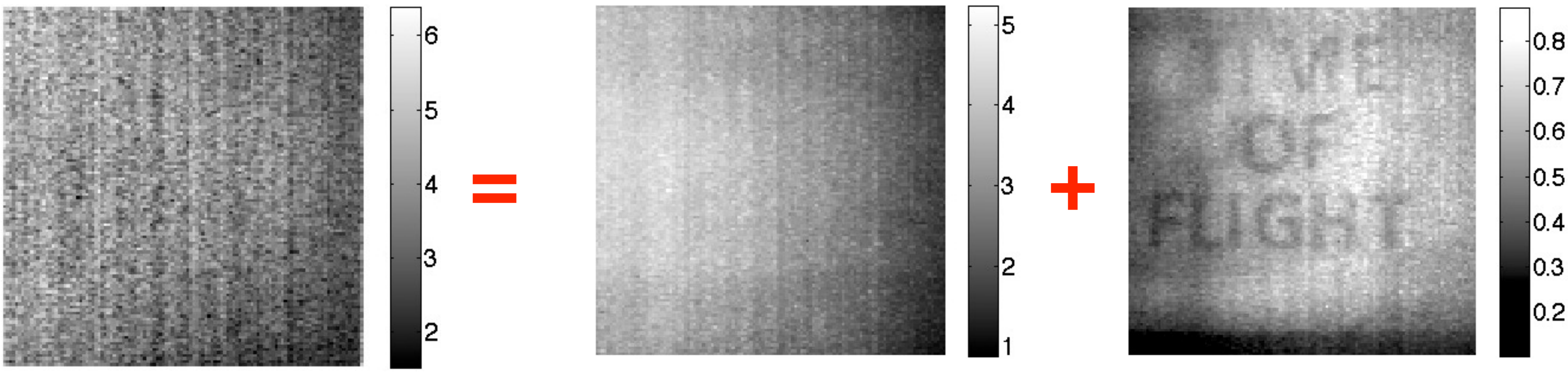
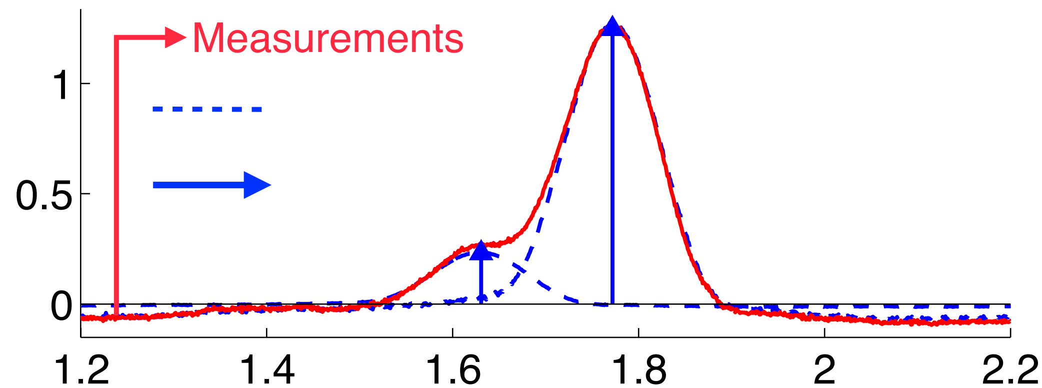
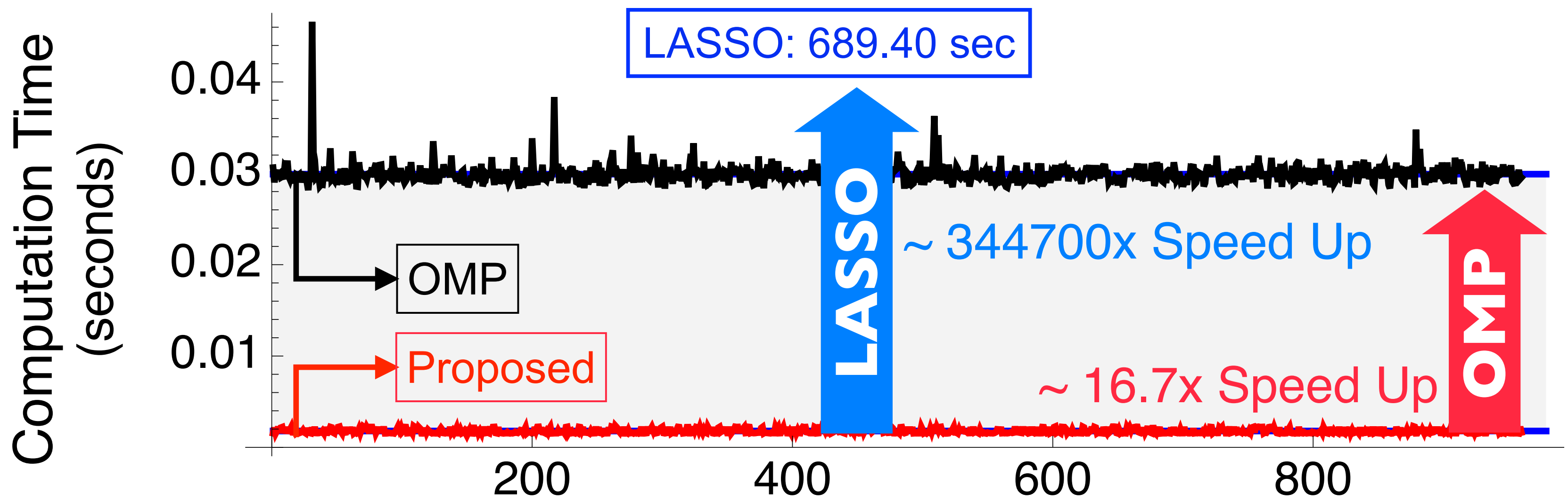
What about Time Domain?



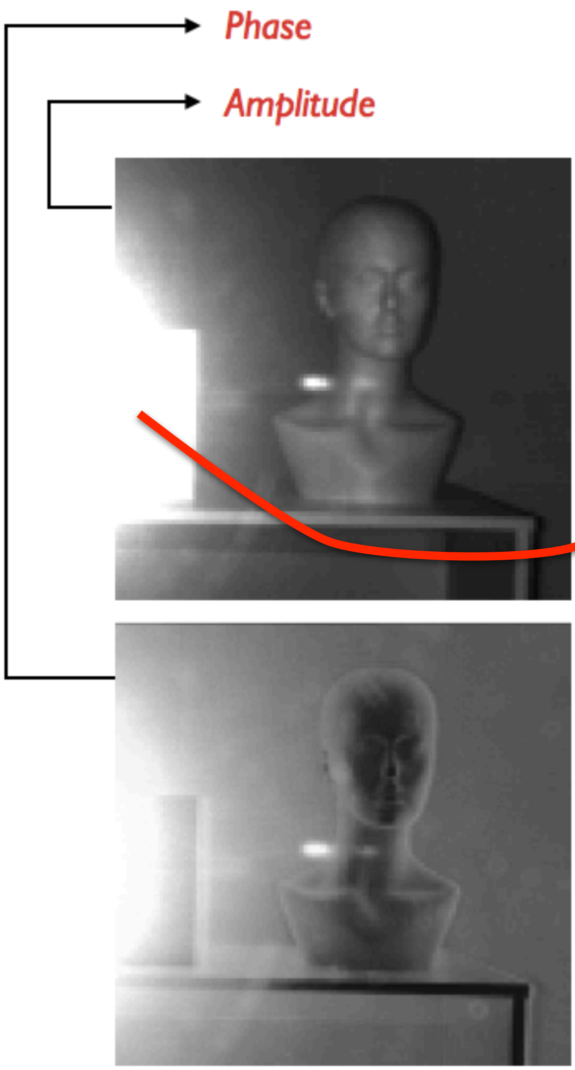
Measurement **Decomposed Depth 1** **Decomposed Depth 2**

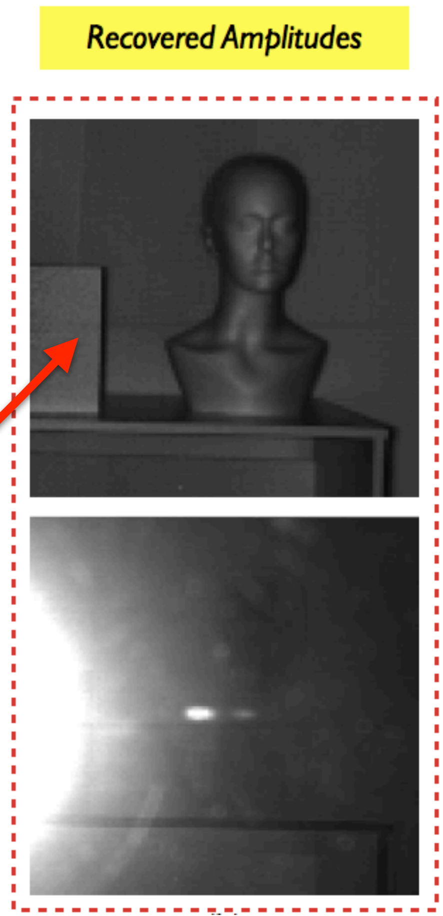


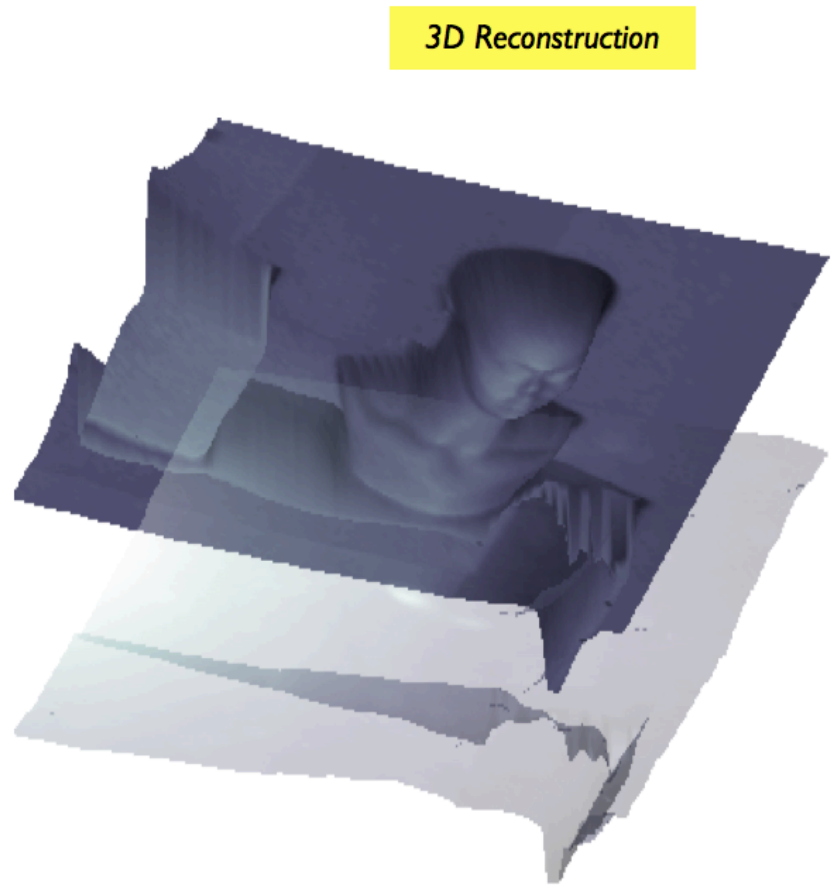
Solving Sparse Problems?



What about Kinect?







*Jointly with **Shahram Izadi**

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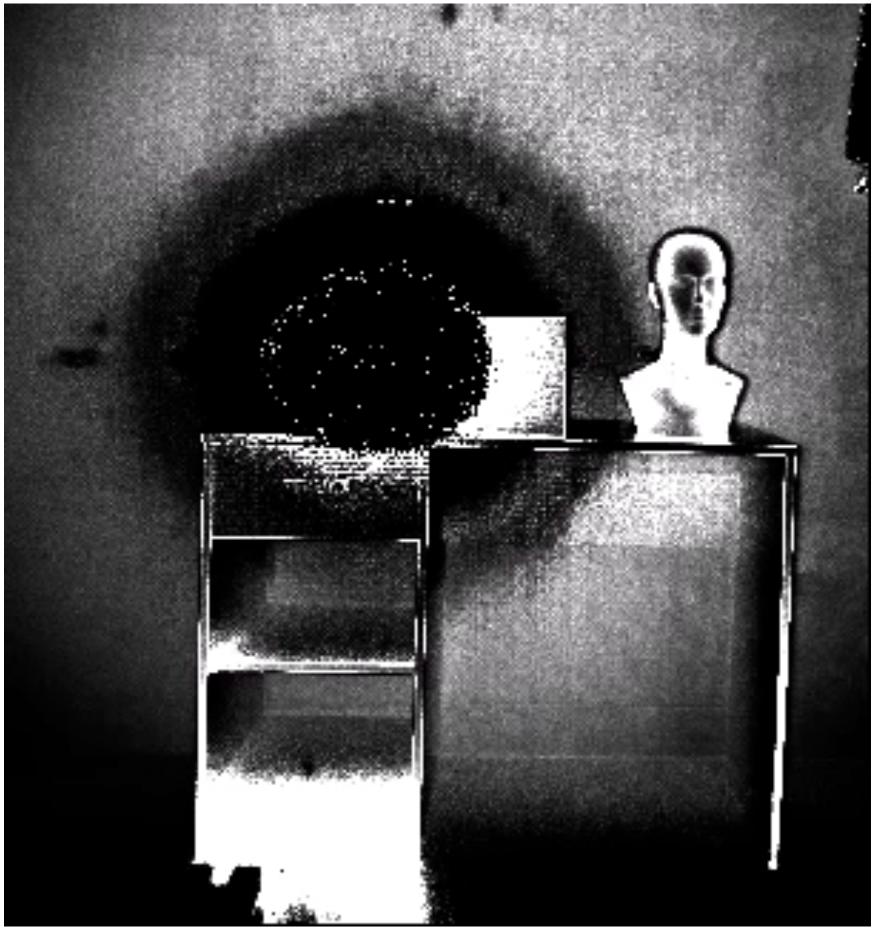
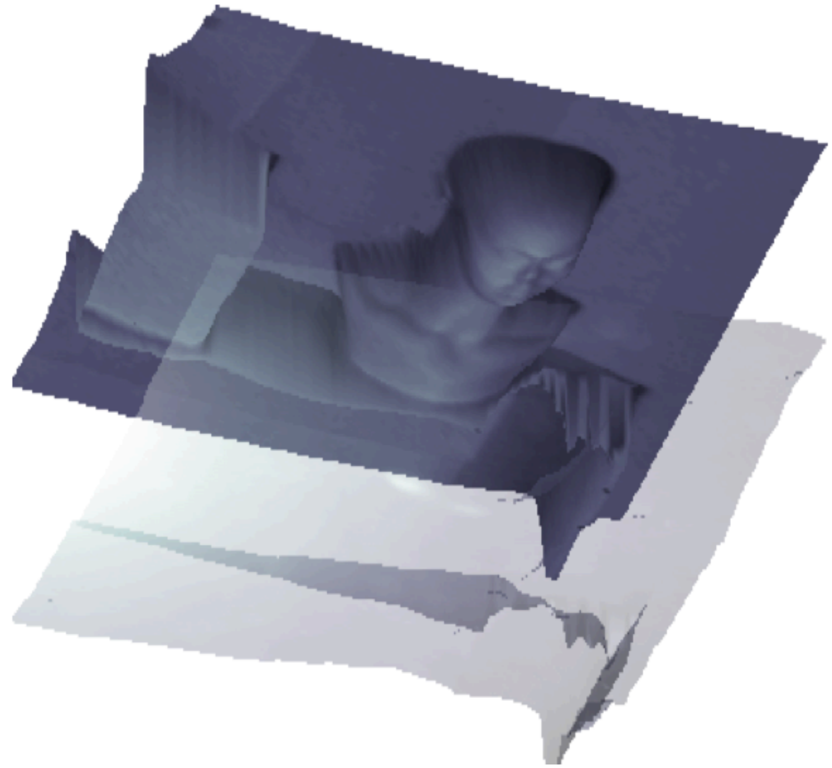
Transient Movies ?





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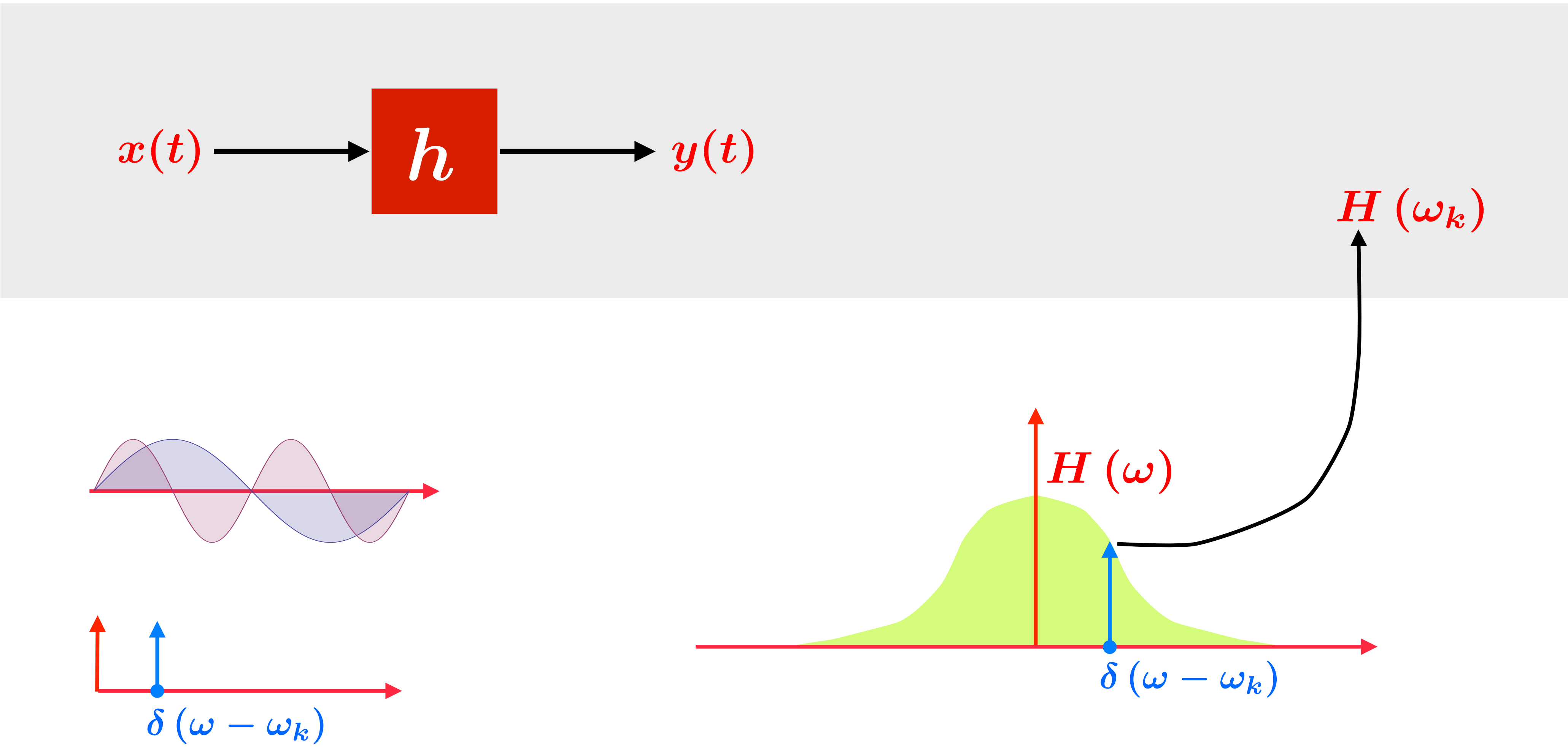
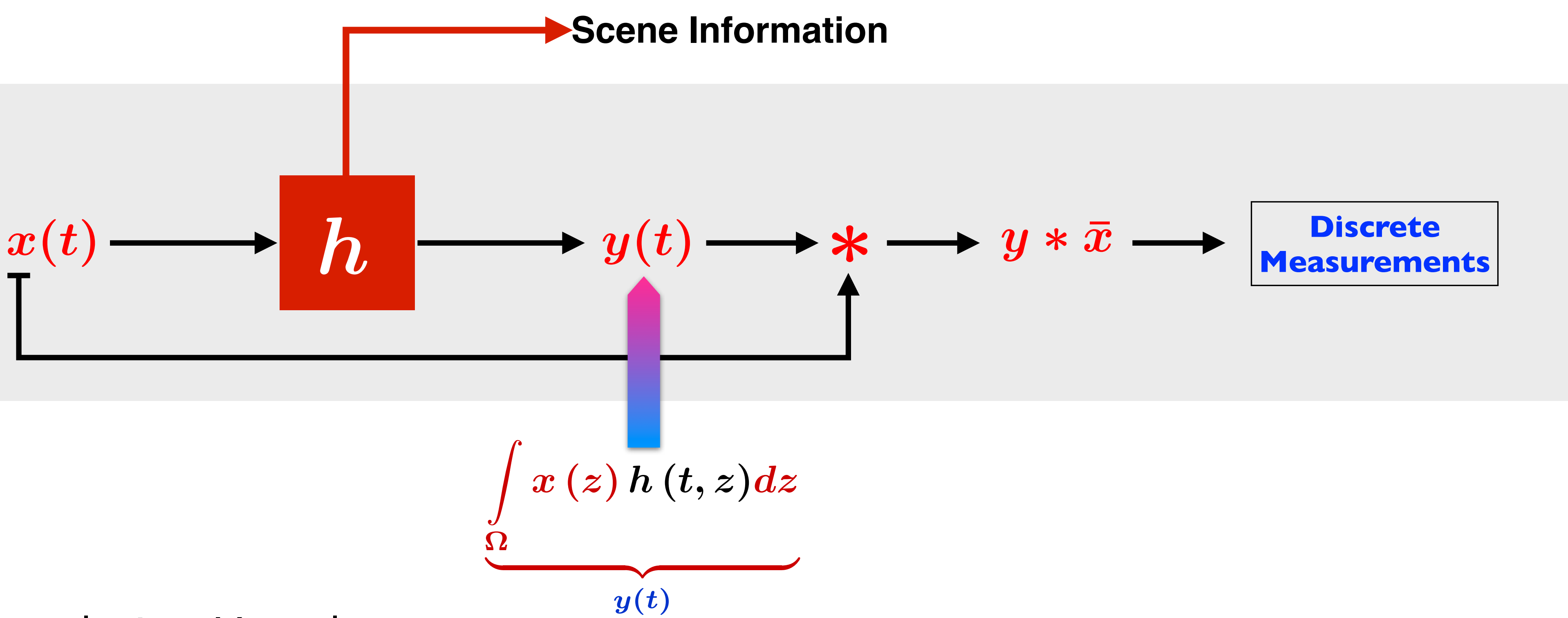
Towards a Conceptual Unification ...

Model Setup: Inspiration from Linear Systems



Model Setup: Fredholm

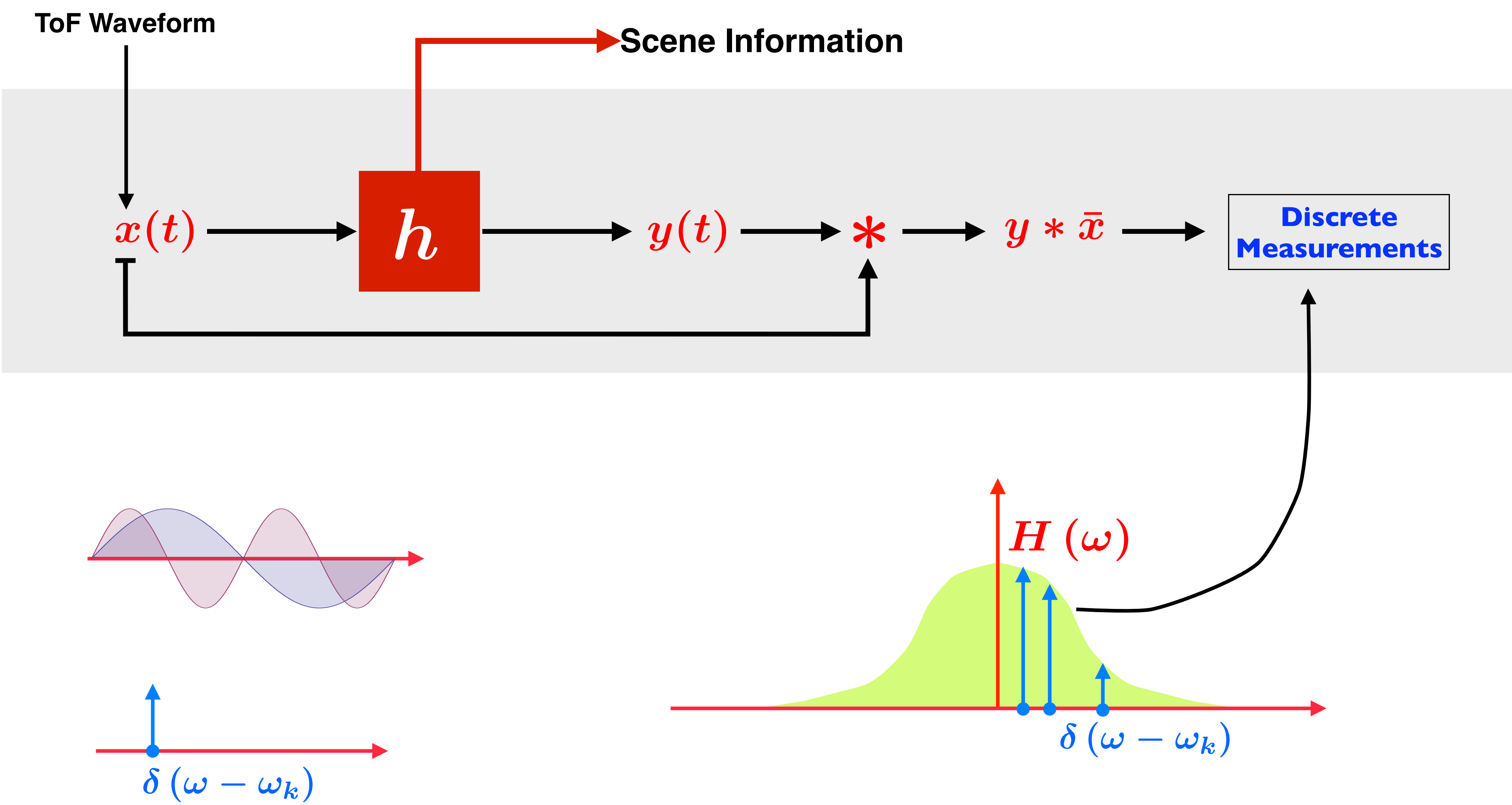


- Convolution Kernels

$$h(t,z) = h(t-z) \Leftrightarrow m(t) = (h * x * \bar{x})(t)$$

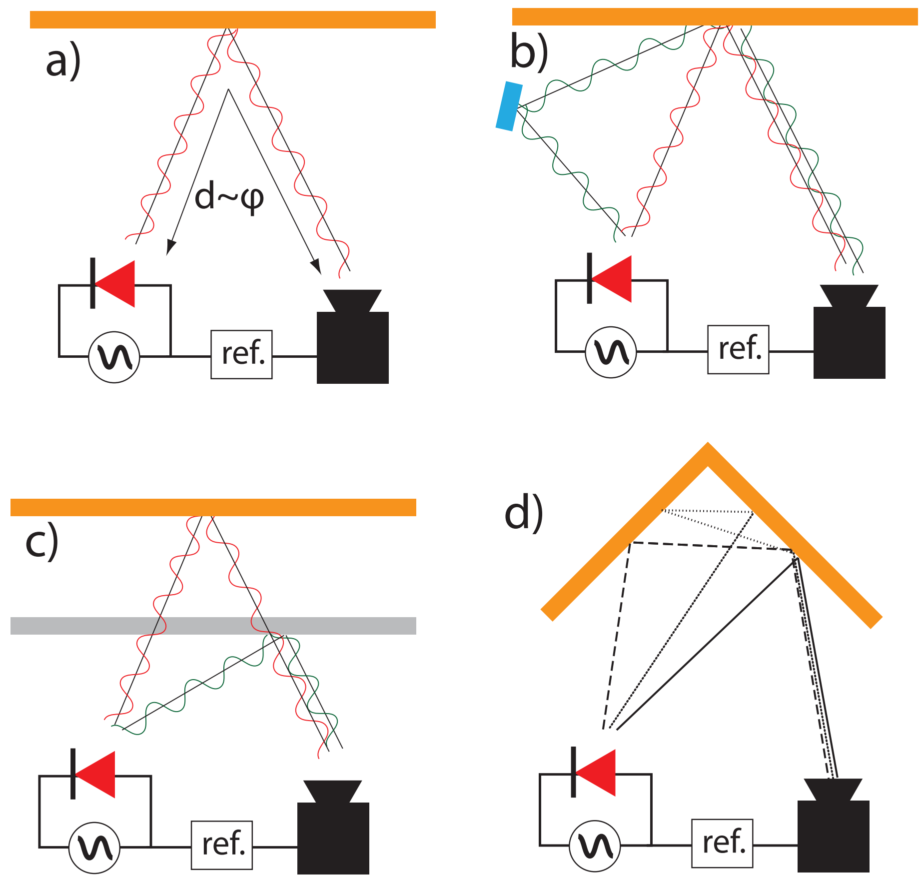
$$\widehat{m}(\omega) = \widehat{h}(\omega)\, |\widehat{x}(\omega)|^2$$

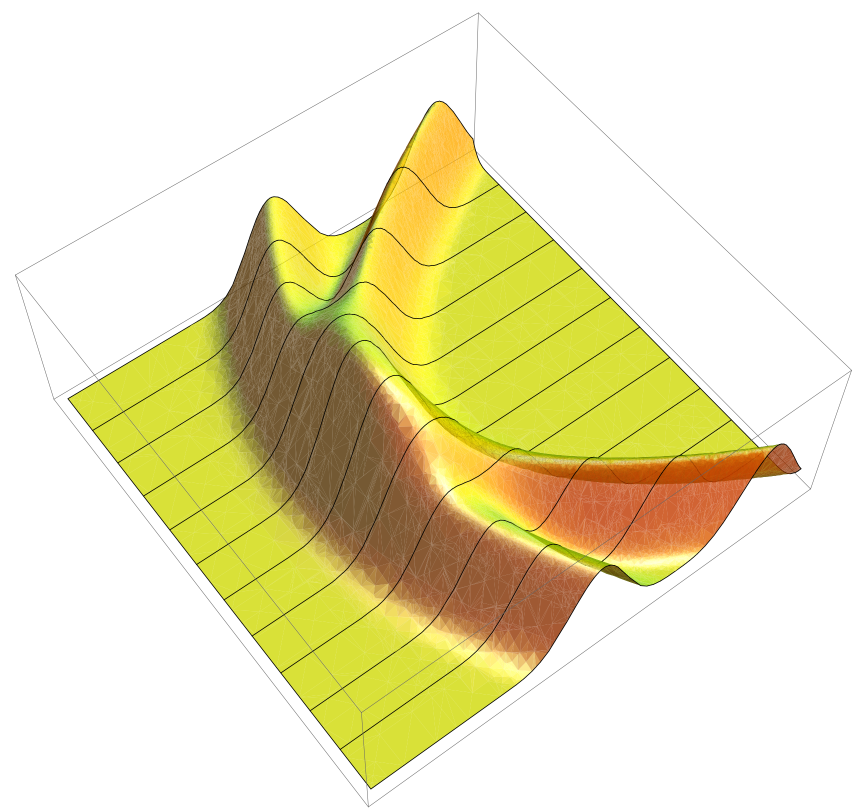
Model Setup: Fredholm



Model Setup: Fredholm — Multi-Path Interference

- Ex.1: Sparse Linear Opertors $h(t) = \sum_{k=0}^{K-1} \mu_k \delta(t - 2d_k\nu)$

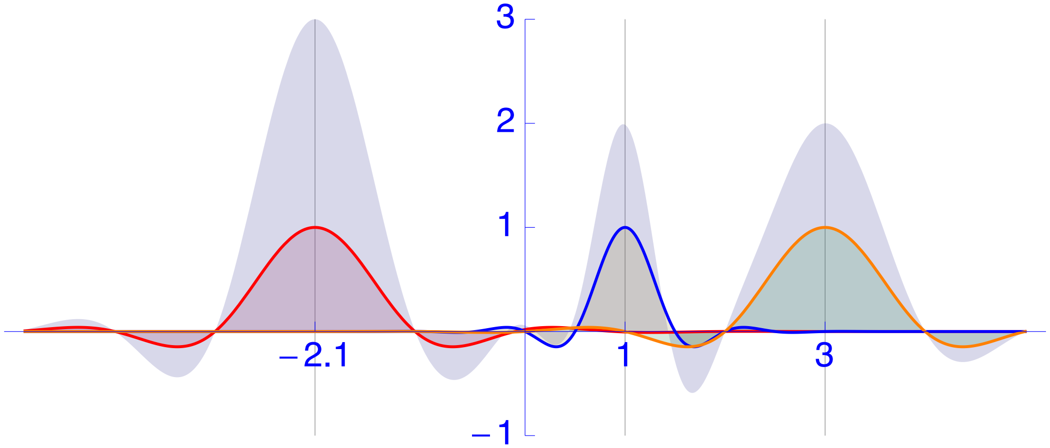


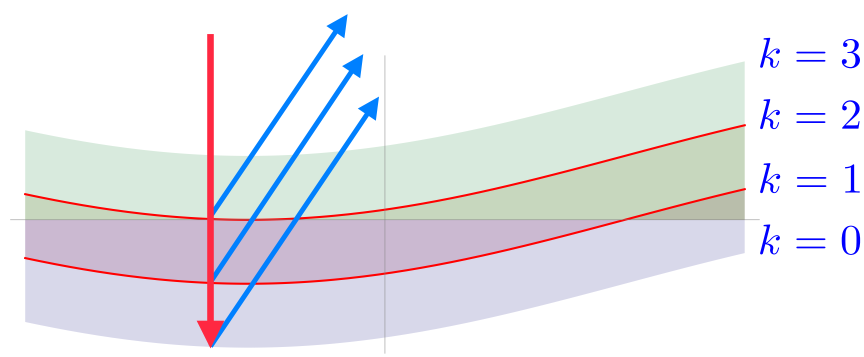


Abstraction of Problem: Degrees of Freedom

Conceptually, we are interested in estimating $(K, \boldsymbol{\alpha}, \boldsymbol{\tau}, \boldsymbol{\gamma})$ from the measurements observed through a filter φ such that:

$$\underbrace{f_{\text{in}}(t)}_{\substack{\text{Probing}\\ \text{Signal}}} \to \boxed{\mathcal{M}_K^{(\alpha,\gamma,\tau)}\{\cdot\}} \to \underbrace{\mathcal{M}_K^{(\alpha,\gamma,\tau)}\{f_{\text{in}}\}}_{\text{Multi–Path Version}} \to \boxed{\varphi(t)} \to \underbrace{\arg \min_{(\hat{\alpha},\hat{\gamma},\hat{\tau})} \left\| f_{\text{in}} - \varphi * \mathcal{M}_K^{(\alpha,\gamma,\tau)}\{f_{\text{in}}\} \right\|_2^2}_{\text{Processing}}.$$





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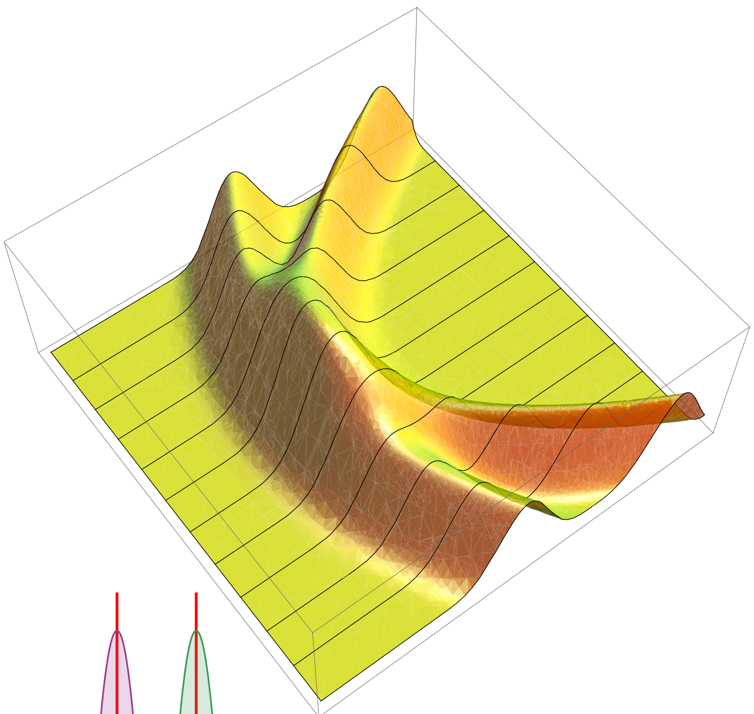
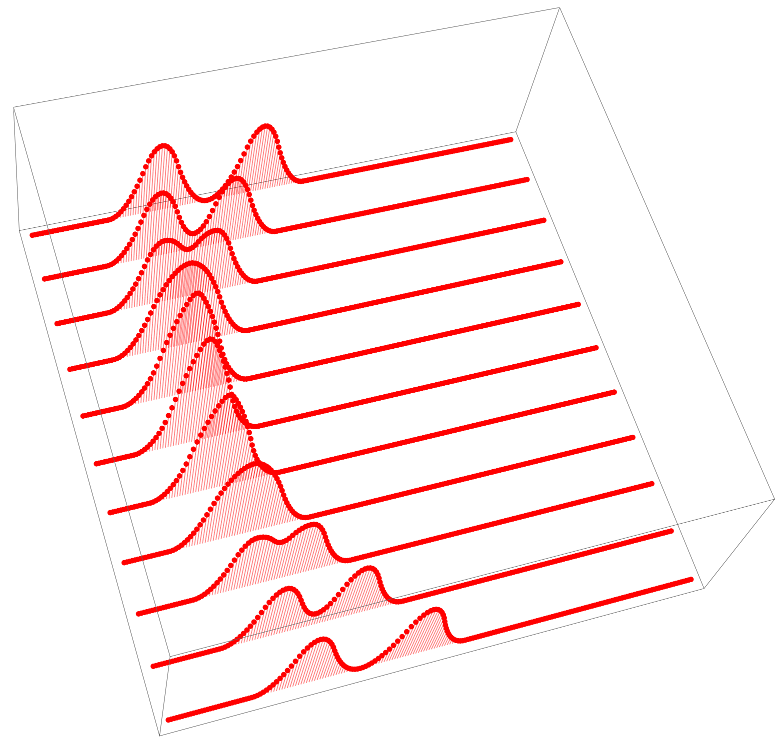
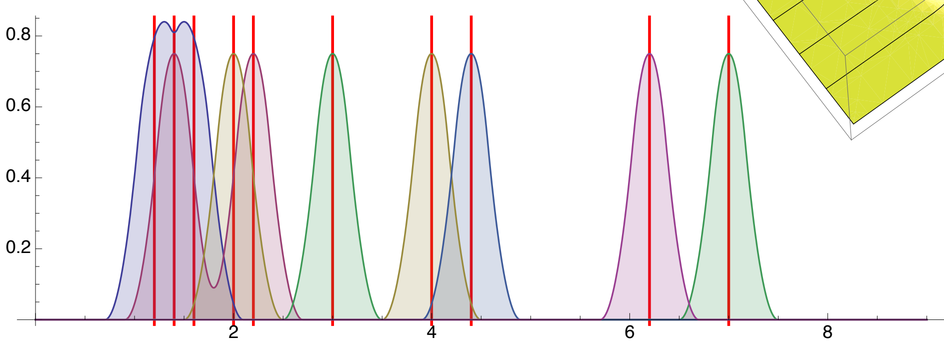
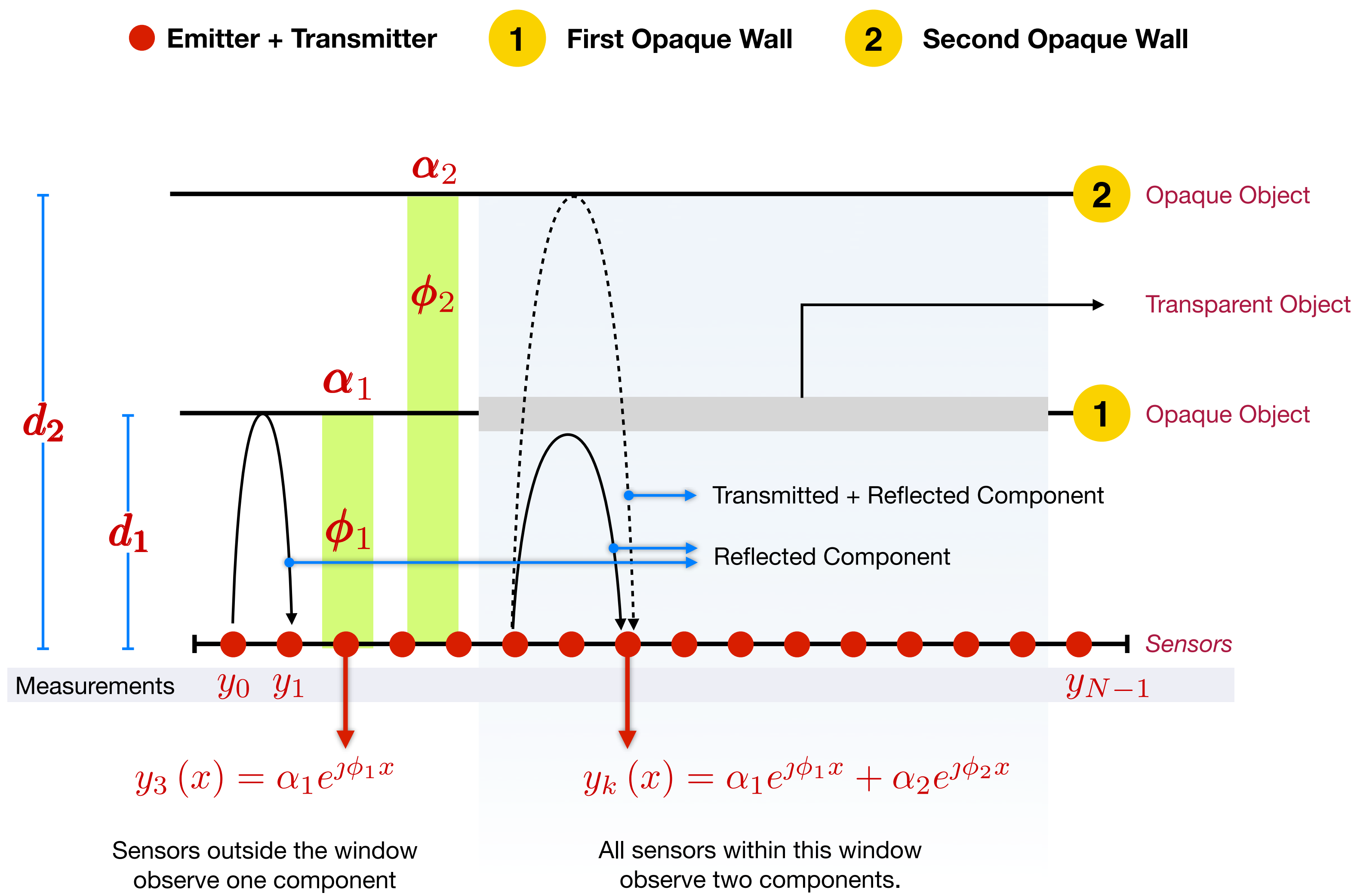


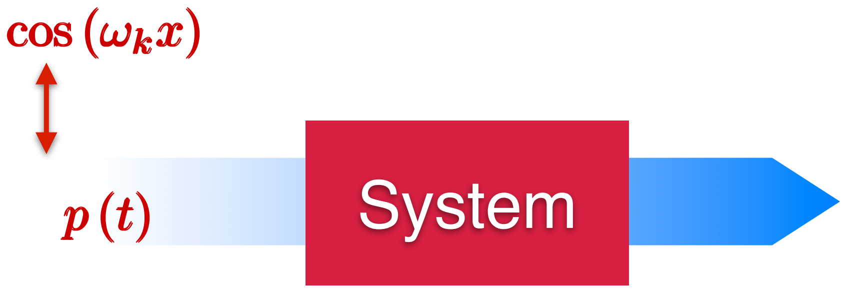




Mixed Pixel Problem at Sensor Level

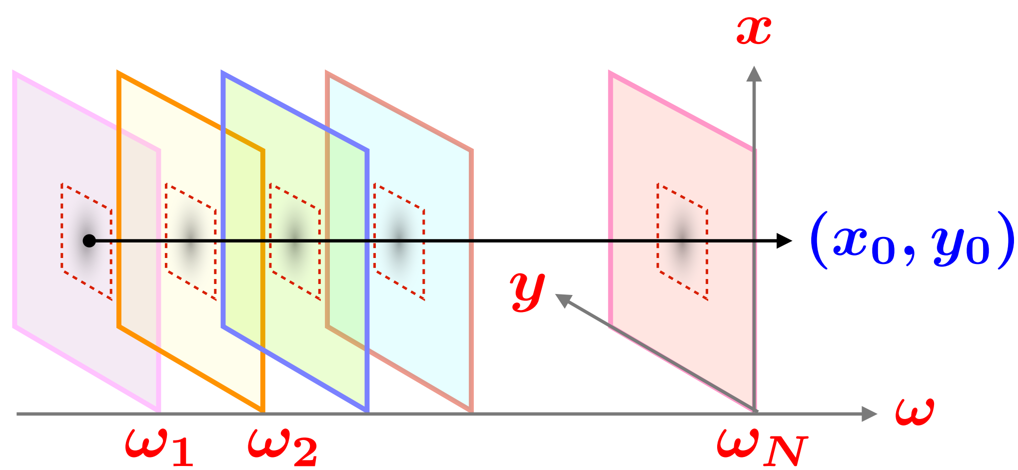


The Multi-Frequency Case: Hypothesis

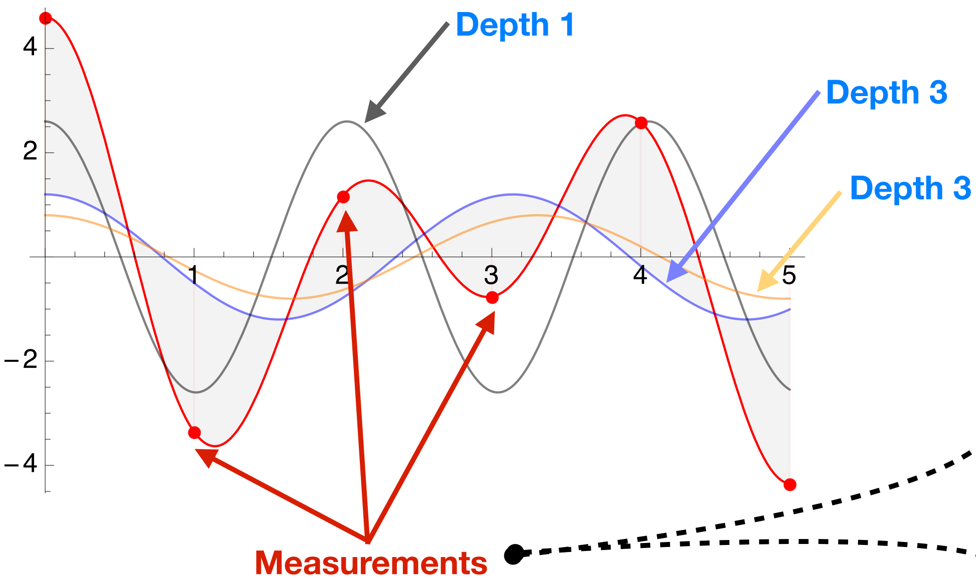


$$p * \sum_{m=0}^{M-1} \mu_m \delta(t - 2d_m \nu) \equiv \sum_{m=0}^{M-1} \mu_m p(t - 2d_m \nu)$$

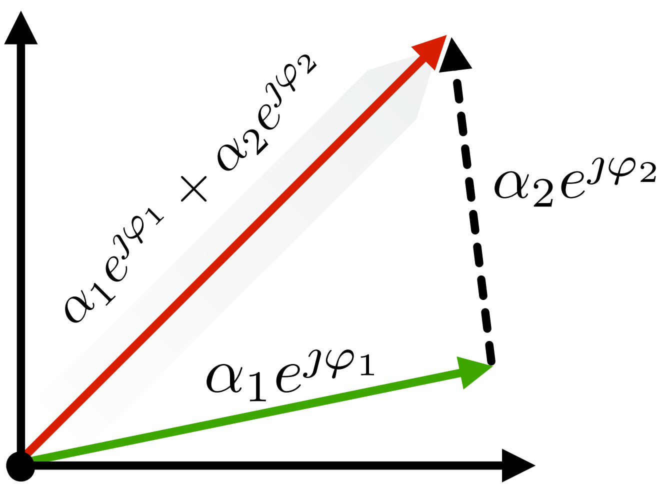
— No inter-reflections $\Rightarrow M$ is finite.
— Static scene or *NO* functional dependence of shifts on time!
— Uniform frequency sampling: Fast Algorithms, Elegant Theory and Nice Bounds!



From Sensor to Forward Model



$$s(x) = \sum_{k=0}^{K-1} \alpha_k e^{j\phi_k x}$$



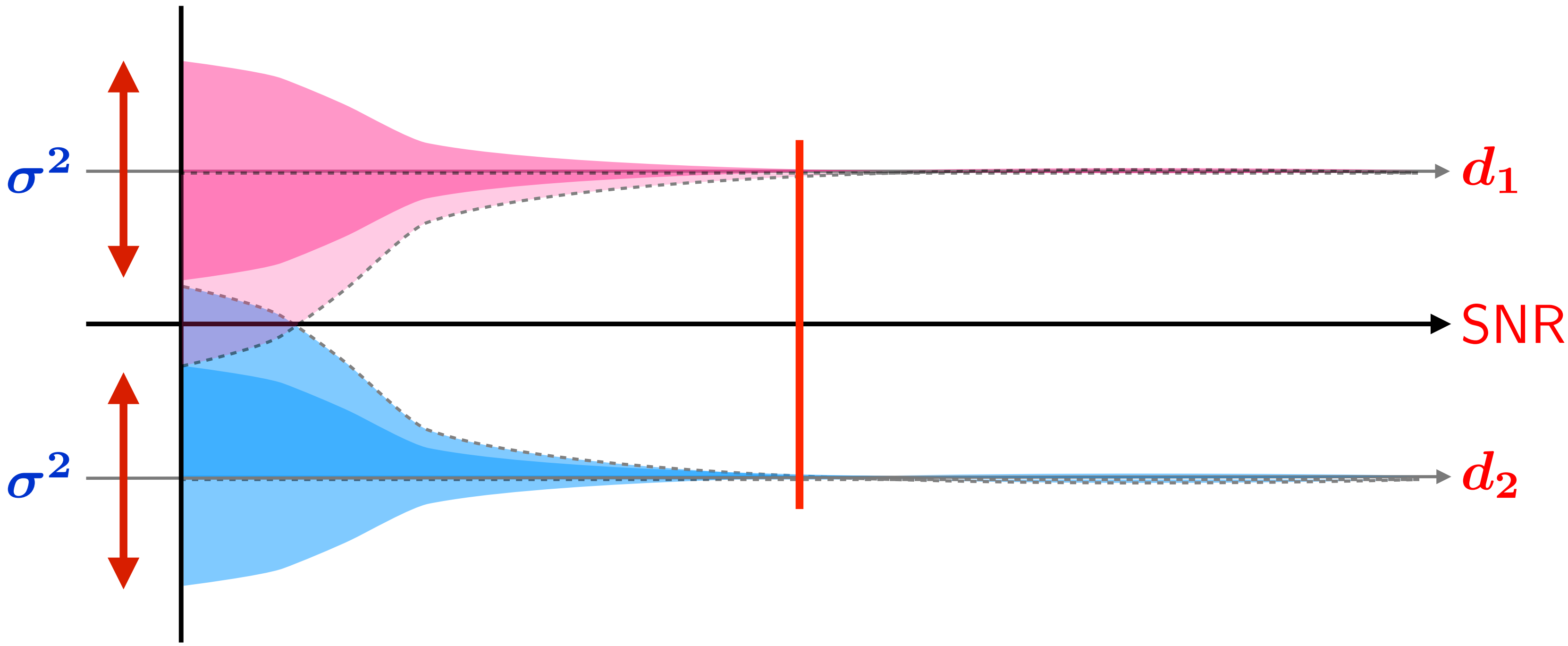
Estimate Matrix | **Weights**

Depth

$$\begin{bmatrix} s[0] \\ \vdots \\ s[K] \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \mu_0 & \mu_1 & \cdots & \mu_{K-1} \\ \vdots & \vdots & \cdots & \\ \mu_0^{K-1} & \mu_1^{K-1} & \cdots & \mu_{K-1}^{K-1} \end{bmatrix} \begin{bmatrix} \alpha[0] \\ \vdots \\ \alpha[K] \end{bmatrix}$$

$$\mu_k = e^{j\phi_k}$$

Bounds for Depth Resolution



$$\Delta d \geqslant \frac{D}{2\pi}\sqrt{\frac{S_m}{N}}\frac{1}{\sqrt{\mathrm{PSNR}}}$$

Key References

- A. Bhandari, A. Kadambi, and R. Raskar, "Sparse Linear Operator Identification without Sparse Regularization," *Proc. of ICASSP*, May 2014.

- A. Bhandari, A. Kadambi, R. Whyte, C. Barsi, M. Feigin, A. Dorrington, and R. Raskar, "Resolving multipath interference in time-of-flight imaging via modulation frequency diversity and sparse regularization," *Optics Letters*, vol. 39, no. 7, 2014.

- A. Kadambi, A. Bhandari, R. Whyte, A. Dorrington, and R. Raskar, "Multiplexing illumination via low cost sensing and nanosecond coding," in *IEEE International Conference on Computational Photography (ICCP)*, March 02-04 2014.

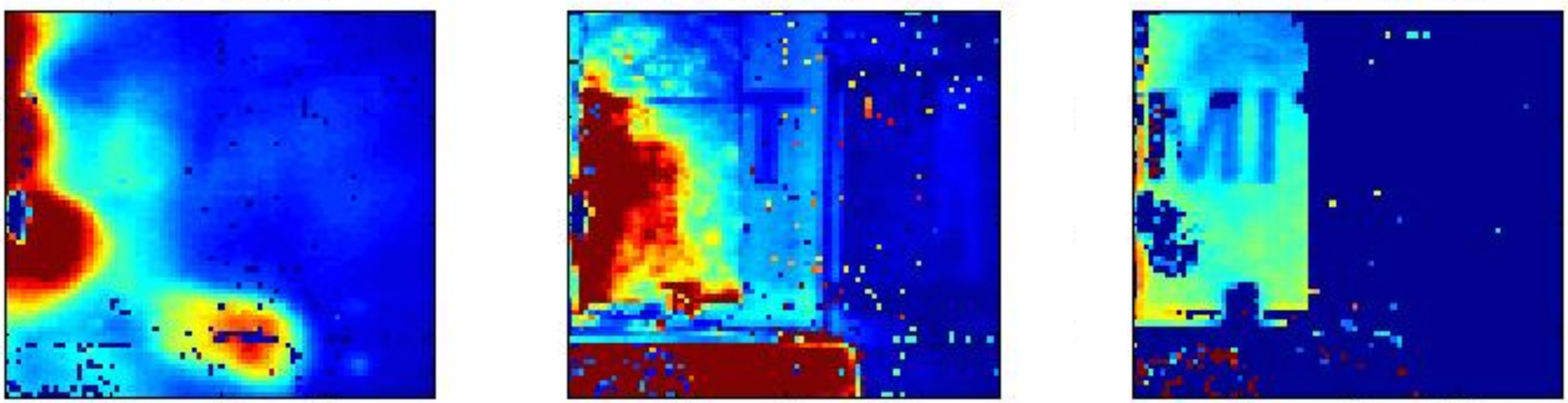
- A. Kadambi, R. Whyte, A. Bhandari, L. Streeter, C. Barsi, A. Dorrington, and R. Raskar, "Coded time of flight cameras: Sparse deconvolution to address multipath interference and recover time profiles," in *SIGGRAPH Asia 2013*.

Conclusion

— Provide a unified perspective for inverse problems in ToF imaging.

— Develop a Toolbox that is applicable to wide variety of applications for ToF Imaging.

— Demonstrate practicability of signal processing algorithms in context of ToF Imaging.



Example of Reconstruction