AMUEM 2009 – International Workshop on Advanced Methods for Uncertainty Estimation in Measurement
Bucharest, Romania, 6-7 July 2009

Graphical Modelling of Measurement Uncertainties in Vision-Based Metrology

Markus Brandner
Institute of Electrical Measurement and Measurement Signal Processing
Graz University of Technology
Graz, Austria
Email: brandner@ieee.org

***Abstract*—Measurement systems perform a quantitative comparison of an unknown physical quantity with a known reference. Vision sensors used in metrological applications provide a non-intrusive and non-invasive way to estimate geometric measurands and are, therefore, well suited for many industrial applications. In recent years the availability of high-resolution sensors and adequate processing power has led to an increased importance of vision-based measurement applications. This paper is concerned with the evaluation of measurement uncertainties in vision-based applications. In particular, we discuss the applicability of Gaussian uncertainties in vision-based metrological applications and present a frame-work for the uncertainty propagation of Gaussian quantities. The frame-work includes a guideline to model the measurement process based on the cause-effect diagram using simple graphical building blocks.**

I. Introduction

Efforts have been undertaken in metrology in order to develop a general frame-work that can be used to identify the quantity of the measurand and to provide means to judge on the quality of this result. These developments led to the introduction of the *Guide to the Expression of Uncertainty in Measurement* (GUM, [1]). The foremost aim of the GUM developments was to provide a recommendation for the treatment of measurement uncertainty that is *universal*, *internally consistent*, and *transferable* [1].

The standard GUM extensively uses the concept of *degrees of freedom* to fuse information from different sources. This concept is a constant point of criticism in the literature (cf. Lira [2]). In particular, the fusion of quantities derived using statistical methods (e.g. averaging over a number of measurements) with quantities denoting an expert opinion (e.g. prior knowledge about interval boundaries) are not satisfactorily covered by the GUM proposal. Weise and Wöger [3] and later Kacker and Jones [4] resort to the consistent use of Bayesian statistics in the context of uncertainty computations. Both approaches remove an inconsistency in the GUM interpretation of coverage probabilities. Kacker and Jones [4] provide a modified set of rules based on the GUM recommendations that are built upon the Taylor approximation of the measurement equation. Their proposed modifications cover the propagation of first and second order moments neglecting modifications of the underlying distributions. Weise and Wöger [3] instead propagate distributions providing a frame-work that is more generally applicable.

Different approaches to the treatment of uncertainty in the domain of computer vision have been reported in the literature [5]–[8]. Using the central limit theorem (CLT, cf. e.g. [9]) as a key argument, the use of the Gaussian assumption is suggested by many researchers. Heuel [10] and Criminisi [11] show that Gaussian densities can be applied to represent homogeneous geometric entities. In an earlier work [12] we apply first order propagation of Gaussian quantities to a vision-based tracking application. Based on the exclusive use of Gaussian densities to describe both physical quantities and prior information, the Bayesian extensions of the GUM document are easily implemented in analytic form.

Sommer and Siebert [13] propose a systematic solution to the model building problem in metrology. The authors use three building blocks to identify and visualise influencing factors and uncertainty contributions. Based on the cause-and-effect approach, *Parameter Sources*, *Transmission Units*, and *Indicating Units* are employed to obtain the measurement equation. This equation is then reversed to obtain the GUM-compliant model equation. Finally, the measurement uncertainty of the unknown quantity can be derived.

In this work we formulate a frame-work for the propagation of Gaussian uncertainties in the context of vision-based metrology. All processing steps can be carried out analytically, thus avoiding any simulation-based computations with the potential lack of real-time performance. The frame-work is consistent with the Bayesian extensions to the standard GUM. Using a number of simple building blocks we propose simple modelling steps to analytically derive the measurement uncertainty in vision-based applications explicitly covering inter-parameter dependencies.

II. Uncertainty in Vision-based Metrology

Our discussion follows the flow of information within a typical vision-based metrology application as outlined in Figure 1: The camera maps the scene onto its image plane and acquires a 2D intensity profile $\mathcal{I}$. A feature detector is then used to identify features $\boldsymbol{f}$ (e.g. circular blobs) and to estimate their respective parameters (e.g. blob area and centre of gravity). These parameters are then being further processed by means of a transformation in order to obtain the best estimate of the unknown parameter $\hat{\boldsymbol{\theta}}$. Using this example we

978-1-4244-3593-7/09/$25.00 ©2009 IEEE

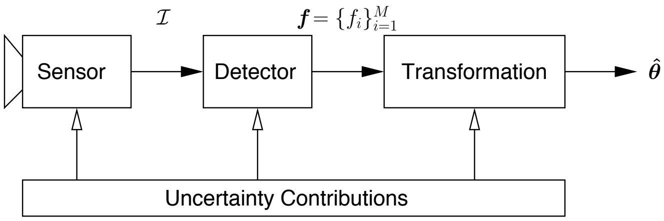


Fig. 1. Components of a typical vision-based measurement system. The best estimate of the unknown quantity $\hat{\boldsymbol{\theta}}$ is obtained given an 2D intensity profile $\mathcal{I}$ acquired by the sensor.

identify the following requirements on a frame-work for the treatment of uncertainties:

1) *Treatment of multivariate measurands*
2) *General applicability to geometric entities*
3) *Common handling of Type A and Type B uncertainties*
4) *Propagation through different processing blocks*
5) *Real-time performance*
6) *Handling of 'single measurement' scenarios*
7) *Proper treatment of statistical dependencies*

Combining the ideas of uncertain projective geometry (cf. Heuel [10]) with the GUM/Bayes approach to the treatment of measurement uncertainty allows us to simplify and unify the modelling process for problems in geometric metrology. In summary our approach covers the following situations:

- Homogeneous entities represented by Gaussian random vectors. For example, a homogeneous point in 2D is given by
$$\underline{\mathbf{x}} \sim \mathcal{N}(\mathbf{x}, \Sigma_{\underline{\mathbf{x}}\underline{\mathbf{x}}}). \tag{1}$$
- Mapping of homogeneous entities including the propagation of parameter uncertainties. Mapping functions include: geometric transformations such as translations, rotations, and perspective mappings. Further, geometric construction (e.g. point results from intersecting two lines) and Euclidean and spherical normalisation are covered.
- Measurement updates of geometric entities with prior knowledge based on Gaussian random vectors.
- Correlations between geometric entities.

A. Nomenclature

In the subsequent modelling process, we will use a unified nomenclature which assigns underlined symbols to quantities in a metrological sense. Their corresponding non-underlined version is used to denote realisations of the quantity. If it is clear from the context, we will also use the non-underlined symbols to denote the best estimates of the corresponding quantities. Thus, $\underline{c}$ is a scalar quantity and c is the corresponding realisation or best estimate. We use u_c to denote the standard uncertainty of the best estimate and U_c as expanded uncertainty associated to a given coverage factor k. Similarly, a vector-valued quantity is referred to as $\underline{\mathbf{x}}$. The best estimate of $\underline{\mathbf{x}}$ is given by $\mathbf{x}$. The uncertainty matrix $\mathsf{U}_{\mathbf{x}}$ of $\mathbf{x}$ corresponds to the covariance matrix $\Sigma_{\underline{\mathbf{x}}\underline{\mathbf{x}}}$ of the quantity. It is now straight forward to make explicit the correlation between two different quantities $\underline{\boldsymbol{m}}$ and $\underline{\boldsymbol{n}}$ by means of their cross-covariance matrix $\Sigma_{\underline{\boldsymbol{m}}\underline{\boldsymbol{n}}}$. The multivariate equivalent to the expanded uncertainty is obtained by finding constant density curves of the PDF which correspond to a given coverage probability p. For 2D Gaussian quantities these curves are ellipses of general orientation.

B. Steps of the Modelling Process

The *model equation* expresses the functional relationship between the measurand $\underline{Y}$ and the input quantities $\underline{X}_1, \ldots, \underline{X}_N$. However, the structure of the model equation usually does not directly reflect the processing steps involved in the measurement process. If we assume that the measurand $\underline{Y}$ is determined by reading the result of the quantity $\underline{X}_3$, the model equation can be reformulated such that X_3 is given by

$$\underline{X}_3 = f_M(\underline{Y}, \underline{X}_1, \underline{X}_2, \underline{X}_4, \ldots, \underline{X}_N), \tag{2}$$

which is referred to as the *measurement equation*. Sommer and Siebert [13] suggest to base the model building process on this measurement equation as it physically relates the *cause*, i.e. the measurand $\underline{Y}$, to an *effect*, i.e. the reading $\underline{X}_3$. We propose to perform the following steps in order to evaluate the measurement uncertainty of a vision-based metrology system using this model equation:

Description of the Measurement Task: A complete description of the measurement task is the most important step of the modelling process. This description includes the input quantities and – most importantly – the measurand as well as their mutual statistical dependencies.

Cause-Effect Relations: All quantities included in the above description must be brought into a form following the idea of the cause-effect approach. It is helpful to visualise these relations using a simple graph [13].

Measurement Model: In the next step, the measurement model is derived using cause-effect relations identified in the previous modelling step. The measurement model now relates *indications* or *observations* made by the sensor to the measurand. In most cases, it is not necessary to develop the measurement model in full detail. Rather, a coarse overview of the processing steps involved in the measurement process is sufficient as the next step in the modelling procedure aims at a fully qualified uncertainty model.

Model Equation: The model equation relates all observations and other input quantities to the measurand. This core equation of the metrological system includes all quantities and their respective uncertainties. This step can be simplified by developing a graphical model. Due to the fact that all geometric quantities in our framework are represented by Gaussian random variables and linear transformations thereof are again Gaussian random variables, the graphical model is composed of a small number of building blocks:

- *Source*: Uncertain quantity characterised by its best estimate and the uncertainty matrix. The source block is frequently used to represent prior information.

- *Transformation using constant parameters*: Simple transformations such as scaling functions are covered by this more general class of transformations. The uncertainty of the output quantity is only caused by the uncertainty of the input quantity.
- *Transformation of uncorrelated quantities*: Transformations with stochastic parameters extend the previous building block by the ability to model uncertainty contributions caused by uncertainties of the parameters. Examples for this class of transformations are geometric constructions such as the intersection of two lines resulting in an uncertain point. The lack of correlation between the input quantities is depicted by input quantities that enter the block on different sides or equivalently by small rectangles attached to the input quantities denoting the range of correlated quantities.
- *Transformation of correlated quantities*: As opposed to the previous class of transformations, this block explicitly covers correlations between quantities. Examples of this class of transformations are geometric constructions using entities which are based on a common source of uncertainty. Graphically, correlation is indicated by grouping all correlated input quantities onto the same side of the block.
- *Bayesian update*: This block performs the Bayesian update for input quantities. Prior knowledge is provided by means of Gaussian random variables. Example applications for this building block include the update step of recursive Bayesian filters (e.g. Kalman filter) or Bayesian feature detectors.

In summary, the components of the graphical model and their respective laws for the propagation of uncertainties are shown in Table I. Note that in contrast to [13], no indication block is used in our context. In most vision-based metrology applications digital indications with quantisation steps associated to the resolution of the underlying number format are used. As this resolution is usually much higher than the uncertainties under consideration, we skip this explicit block from the model. Instead, we explicitly visualise statistical dependencies between quantities.

C. Limitations of the Approach

The transformation of Gaussian quantities results in another Gaussian quantity only for linear transformations. As soon as the transformation exhibits a non-linear contribution, the resultant quantity starts to deviate from the Gaussian assumption with the degree of deviation depending on the degree of non-linearity introduced by the transformation function. From the metrological point of view, these deviation from the Gaussian are of concern for the following reasons:

1) Non-linearities cause the PDF of the output quantity to deviate from the Gaussian shape. An example of a non-linear transformation function frequently encountered in vision-based metrological applications is the correction for lens-distortions.
2) The analytic derivations of Bayes' law are only applicable to Gaussian quantities. Any deviation from this Gaussian assumption will lead to approximate solutions and, therefore, to inaccurate uncertainty estimates.
3) Non-linearities introduce a bias of the best estimate of the output quantity. The bias generally is a function of the best estimates of the input quantities as well as of the input uncertainties.

These effects usually strongly dependent on the degree of correlation between the input quantities. The impact on the determination of the measurement uncertainties in such situations is shown in the next section.

III. Examples

In this section we present two examples to illustrate the modelling process and to show the limitations of the linear propagation of Gaussian uncertainties. We start with the development of a graphical model of a vision-based measurement application and report about the impact of a neglected parameter correlation in a simple transformation function.

A. 2D Displacement Measurement

We derive the uncertainty model of a 2D displacement measurement system which reflects the general structure of a vision-based metrological system as shown in Figure 1. The measurement system is part of a creep test apparatus used to obtain material parameters of polymer samples under specific conditions. The experimental setup and the measurement system are explained in more detail in Brandner *et al.* [14]. Our focus in this section is to justify the particular uncertainty model applied for this measurement system.

Figure 2 depicts the geometric sketch of the displacement system. A single camera is used to acquire an image of a scene comprising a planar reference target and a planar sample target. These targets each consists of circular blob features manufactured into a stainless steel sheet by laser marking. By construction of the setup, the two targets are coplanar so that a homography $\hat{\mathsf{H}}$ can be used to relate the image plane of the sensor Π_{Image} to the $(z = 0)$-plane which holds both targets. During the measurement process a single image is used to simultaneously obtain image points corresponding to the reference target and the sample target. Based on these image points, the sensor estimates the parameters of the homography which are then used to reconstruct the 2D displacement of the sample target with respect to the reference target.

Uncertainty Propagation: For this specific measurement application we note that each image point is mapped by the same sensor and detected under the same illumination conditions of the scene. The common sensor calibration and detection conditions introduce a correlation between the points used to estimate the homography and the points which are transformed using the estimated homography.

Figure 3 shows the graphical model relating the input quantities (i.e. the image centres of the blobs) to the measurand (i.e. the position $\underline{t}$ in metric coordinates). This model graphically represents the measurement equation. Note that

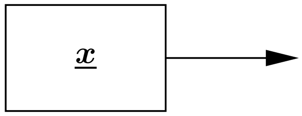
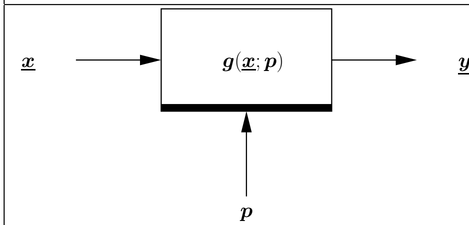
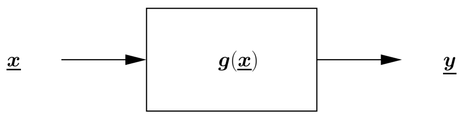
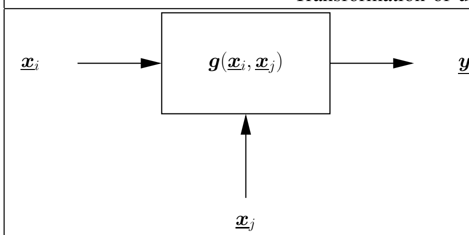
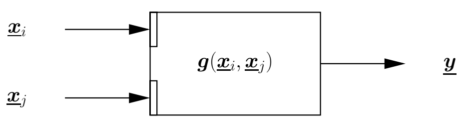
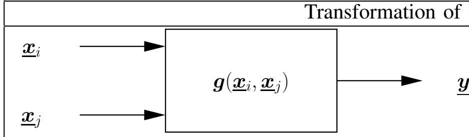
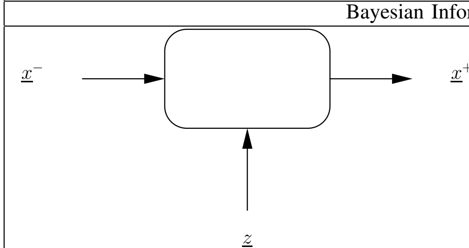
Symbol	Uncertainty Contribution
Source	
	$\Sigma_{\underline{\boldsymbol{x}}\underline{\boldsymbol{x}}}$
Transformation with constant parameters	
 	$\Sigma_{\underline{\boldsymbol{y}}\underline{\boldsymbol{y}}} = \mathsf{J}_g \Sigma_{\underline{\boldsymbol{x}}\underline{\boldsymbol{x}}} \mathsf{J}_g^T$
Transformation of uncorrelated quantities	
 	$\Sigma_{\underline{\boldsymbol{y}}\underline{\boldsymbol{y}}} = \mathsf{J}_{g,\underline{\boldsymbol{x}}_i} \Sigma_{\underline{\boldsymbol{x}}_i\underline{\boldsymbol{x}}_i} \mathsf{J}_{g,\underline{\boldsymbol{x}}_i}^T + \mathsf{J}_{g,\underline{\boldsymbol{x}}_j} \Sigma_{\underline{\boldsymbol{x}}_j\underline{\boldsymbol{x}}_j} \mathsf{J}_{g,\underline{\boldsymbol{x}}_j}^T$
Transformation of correlated quantities	
	$\Sigma_{\underline{\boldsymbol{y}}\underline{\boldsymbol{y}}} = \mathsf{J}_g \begin{bmatrix} \Sigma_{\underline{\boldsymbol{x}}_i\underline{\boldsymbol{x}}_i} & \Sigma_{\underline{\boldsymbol{x}}_i\underline{\boldsymbol{x}}_j} \\ \Sigma_{\underline{\boldsymbol{x}}_j\underline{\boldsymbol{x}}_i} & \Sigma_{\underline{\boldsymbol{x}}_j\underline{\boldsymbol{x}}_j} \end{bmatrix} \mathsf{J}_g^T$
Bayesian Information Update	
	$u^2_{\underline{x}^+} = \left(\frac{1}{u^2_{\underline{x}^-}} + \frac{1}{u^2_{\underline{z}}} \right)^{-1}$ $\underline{x}^+ = u^2_{\underline{x}^+} \left(\frac{\underline{x}^-}{u^2_{\underline{x}^-}} + \frac{\underline{z}}{u^2_{\underline{z}}} \right)$

TABLE I

BUILDING BLOCKS OF THE GRAPHICAL MODEL. UNCERTAINTY CONTRIBUTIONS ARE EXPRESSED BY MEANS OF THEIR RESPECTIVE UNCERTAINTY MATRICES. THE TRANSFORMATION FUNCTIONS $g(\cdot)$ ARE LINEARISED TO THE FIRST ORDER USING THEIR RESPECTIVE JACOBIAN MATRICES J_g.

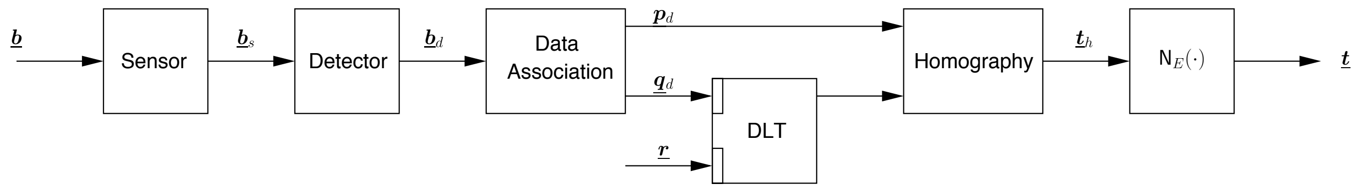


Fig. 3. Uncertainty model of the 2D displacement measurement system. The simultaneous estimation and application of the homography parameters require the proper handling of correlations.

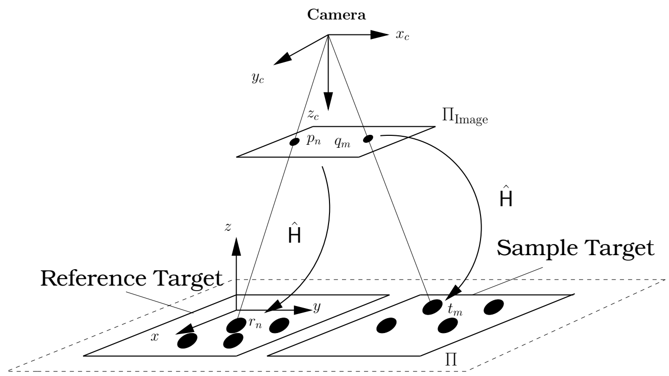


Fig. 2. Outline of the geometry of a single camera/target pair. This *single image acquisition* setup processes the same image twice: First, features on the reference target are used to estimate the transformation parameters $\hat{\mathsf{H}}$. Second, these parameters are applied to features on the sample target in order to estimate the displacement of this target.

we explicitly visualise parameter correlations which is shown by the following two examples: First, the estimation of the homography parameters is performed using the direct linear transform algorithm (DLT, [15]). This algorithm takes as input a sequence of image points, $\underline{\boldsymbol{q}}_d$ and their corresponding model points $\underline{\boldsymbol{r}}$. While the image points are correlated due to their common acquisition conditions, no dependency between the image points and their models is considered in this model. This is denoted by the two rectangles within the DLT block restricting correlations to appear *within* the rectangle only. Second, the final homography parameters are used to map the image point $\underline{\boldsymbol{p}}_d$ in order to obtain $\underline{\boldsymbol{t}}_h$. The common acquisition of $\underline{\boldsymbol{p}}_d$ and $\underline{\boldsymbol{q}}_d$ gives rise to a inter-parameter correlation between the input parameters to the homography block. The rectangle corresponding to the range of correlated input quantities is now extended to all quantities entering the block on the right – and, consequently, omitted for a clear representation.

B. Product of Gaussian Quantities

In the subsequent paragraphs we use the product of two Gaussian random variables to discuss the limitations of the linear propagation of Gaussian uncertainties and the effects of neglecting parameter correlations in more detail. We consider the simple example of a function $g(\cdot)$ which transforms two Gaussian input quantities into their product, i.e.

$$\underline{Z} = g(\underline{X}_1, \underline{X}_2) = \underline{X}_1 \underline{X}_2. \tag{3}$$

We denote the mean values of the input quantities by $\mu_{\underline{X}_1}$ and $\mu_{\underline{X}_2}$, the standard uncertainties by $u_{\underline{X}_1}$ and $u_{\underline{X}_2}$ and the correlation coefficient by ρ. Using these abbreviations the joint density characterising the input quantities is given by

$$f_{\underline{\boldsymbol{X}}}(\boldsymbol{X}) = \frac{1}{2\pi\sqrt{|\Sigma_{\underline{\boldsymbol{X}}\underline{\boldsymbol{X}}}|}} e^{-\frac{1}{2}(\underline{\boldsymbol{X}}-\boldsymbol{\mu}_{\underline{\boldsymbol{X}}})\Sigma^{-1}_{\underline{\boldsymbol{X}}\underline{\boldsymbol{X}}}(\underline{\boldsymbol{X}}-\boldsymbol{\mu}_{\underline{\boldsymbol{X}}})^T} \tag{4}$$

where the vectors $\underline{\boldsymbol{X}} = (\underline{X}_1, \underline{X}_2)^T$ and $\boldsymbol{\mu}_{\underline{\boldsymbol{X}}} = (\mu_{\underline{X}_1}, \mu_{\underline{X}_2})^T$ denote the vector valued input quantity and its mean, respectively. The uncertainty matrix $\Sigma_{\underline{\boldsymbol{X}}\underline{\boldsymbol{X}}}$ expands to

$$\Sigma_{\underline{\boldsymbol{X}}\underline{\boldsymbol{X}}} = \begin{pmatrix} u^2_{\underline{X}_1} & \rho u_{\underline{X}_1} u_{\underline{X}_2} \\ \rho u_{\underline{X}_1} u_{\underline{X}_2} & u^2_{\underline{X}_2} \end{pmatrix}. \tag{5}$$

We are now interested in the uncertainty of the resultant quantity Z as well as in any bias introduced by the transformation function $g(\cdot)$. We observe that the transformation is bilinear in the input quantities. As opposed to strictly linear conditions, the density of the resultant quantity $\underline{Z}$ deviates from the Gaussian density depending on the means and uncertainties of $\underline{\boldsymbol{X}}$. The bias is defined as the difference between the estimated mean and the undisturbed result of the transformation function. Thus, we obtain the bias of $\underline{Z}$ as

$$b(\underline{Z}) = E\{\underline{Z}\} - g(\mu_{\underline{X}_1}, \mu_{\underline{X}_2}). \tag{6}$$

Analytically, the first order moment of the quantity $\underline{Z}$ is given by

$$E\{\underline{Z}\} = \mu_{\underline{X}_1}\mu_{\underline{X}_2} + \rho u_{\underline{X}_1} u_{\underline{X}_2}, \tag{7}$$

and, consequently, the bias is given by

$$b(\underline{Z}) = \rho u_{\underline{X}_1} u_{\underline{X}_2}, \tag{8}$$

indicating a linear dependency on both the correlation coefficient an the product of the contributing standard uncertainties.

Assuming a common relative standard uncertainty of both input quantities, we directly observe the quadratic dependency between $b(\underline{Z})$ and the product of the contributing input uncertainties. The amplification property of the correlation coefficient is visualised in Figure 4. Using relative standard uncertainties of the input quantities, i.e.

$$u_{\underline{X}_1,\text{relative}} = \frac{u_{\underline{X}_1}}{\mu_{\underline{X}_1}}, \tag{9}$$

we find a general upper bound for the bias of $g(\cdot)$ over the full range of correlation coefficients by

$$b_{\max}(\underline{Z}) = u_{\underline{X}_1,\text{relative}} u_{\underline{X}_2,\text{relative}} \mu_{\underline{X}_1}\mu_{\underline{X}_2}. \tag{10}$$

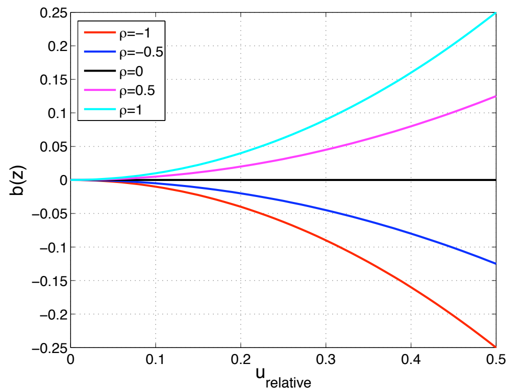


Fig. 4. Estimation bias introduced to the product of two dimensionless scalar quantities as a function of different levels of input uncertainties. The correlation coefficient scales the output uncertainty due to its amplification property.

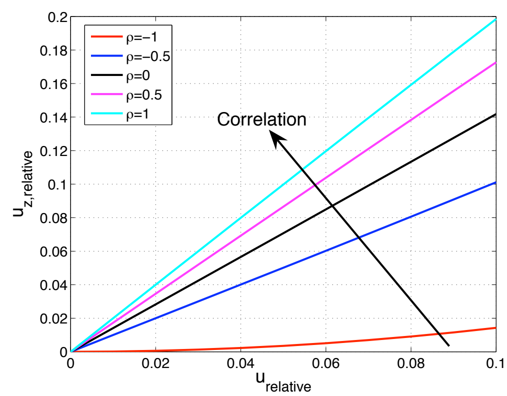


Fig. 5. Estimated relative output standard uncertainty of the bilinear model $\underline{Z} = \underline{X}_1 \underline{X}_2$ as a function of the relative input standard uncertainty. Again the correlation coefficient significantly impacts on the uncertainty of the result.

The standard uncertainty of the output quantity is given by

$$u_{\underline{Z}} = \sqrt{\mu_{\underline{X}_1}^2 u_{\underline{X}_2}^2 + \mu_{\underline{X}_2}^2 u_{\underline{X}_1}^2 + 2\rho\mu_{\underline{X}_1}\mu_{\underline{X}_2}u_{\underline{X}_1}u_{\underline{X}_2} + \rho^2 u_{\underline{X}_1}^2 u_{\underline{X}_2}^2}, \quad (11)$$

which simplifies for uncorrelated input quantities to

$$u_{\underline{Z}} = \sqrt{\mu_{\underline{X}_1}^2 u_{\underline{X}_2}^2 + \mu_{\underline{X}_2}^2 u_{\underline{X}_1}^2}. \quad (12)$$

Figure 5 depicts the resultant standard uncertainty of $\underline{Z}$ for a setup with $\boldsymbol{\mu}_{\underline{\boldsymbol{X}}} = (1,1)^T$ and $u_{\underline{X}_1,\text{relative}} = u_{\underline{X}_2,\text{relative}} = u_{\underline{X},\text{relative}}$. It is important to note that the correlation coefficient has a strong impact on $u_{\underline{Z},\text{relative}}$: while for negatively correlated input quantities, i.e. $\rho \approx -1$, the resultant standard uncertainty is significantly smaller than the input uncertainties, a strong positive correlation characterised by $\rho \approx +1$ amplifies the resultant standard uncertainty. For the uncorrelated case $u_{\underline{Z},\text{relative}}$ takes on a value of $u_{\underline{Z},\text{relative}} \approx \sqrt{2}u_{\underline{X},\text{relative}}$.

We conclude that neglecting the correlations of the input quantities results in two effects: First, a systematic bias is introduced in the result $\underline{Z}$. The upper bound of this bias is given by Equation 10. Second, the standard uncertainty $u_{\underline{Z}}$ is over- or underestimated as shown in Figure 5. Again, upper bounds can be derived given that the input uncertainties are known.

IV. Summary

In this work we discuss a graphical approach to the modelling of uncertainties in vision-based metrology. The presented approach is based on the Bayesian extensions of the GUM and the consistent use of Gaussian densities to describe both the state of knowledge of different quantities and prior information. Our approach extends the state of the art by an explicit visualisation of statistical dependencies in the graphical model. This is in particular useful for measurements based on *single image acquisitions*, which frequently result in correlations between estimated quantities. We discuss the limitations of the modelling approach and present a modelling example using the proposed building blocks.

References

[1] *Guide to the Expression of Uncertainty in Measurement*. International Organization for Standardization (ISO), 1995, iSBN 92-67-10188-9.
[2] I. Lira, *Evaluating the Measurement Uncertainty*, ser. Series in Measurement Science and Technology, M. Afsar, Ed. Bristol and Philadelphia: Institute of Physics Publishing, 2001.
[3] K. Weise and W. Wöger, *Meßunsicherheit und Meßdatenauswertung*. Wiley-VCH, 1999, iSBN 3-527-29610-7.
[4] R. Kacker and A. Jones, "On use of Bayesian statistics to make the Guide to the Expression of Uncertainty in Measurement consistent," *Metrologia*, vol. 40, pp. 235–248, 2003.
[5] D. I. Havelock, "Geometric precision in noise-free digital images," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 11, no. 10, pp. 1065–1075, October 1989.
[6] T. Kanungo, M. Jaisimha, J. Palmer, and R. Haralick, "A methodology for quantitative performance evaluation of detection algorithms," *IEEE Transactions on Image Processing*, vol. 4, no. 12, pp. 1667–1674, December 1995.
[7] B. Triggs, "Joint feature distributions for image correspondence," in *Proceedings of the International Conference on Computer Vision*, 2001, pp. 101–108.
[8] B. Ochoa and S. Belongie, "Covariance propagation for guided matching," in *Statistical Methods in Multi-Image and Video Processing*, Graz, Austria, May 2006.
[9] A. Papoulis and S. U. Pillai, *Probability, random variables, and stochastic processes*, 4^{th} ed. McGraw-Hill, 2002.
[10] S. Heuel, *Uncertain Projective Geometry: Statistical Reasoning for Polyhedral Object Reconstruction*, ser. Lecture Notes in Computer Science. Springer, 2003, no. 3008.
[11] A. Criminisi, *Accurate Visual Metrology from Single and Multiple Uncalibrated Images*. Springer, 2001.
[12] M. Brandner, "Uncertainty estimation in a vision-based tracking system," in *Proceedings of the IEEE Intl. Workshop on Advanced Methods for Uncertainty Estimation in Measurement (AMUEM 2006)*, Sardagna, Italy, April 2006, pp. 40–45.
[13] K. D. Sommer and B. R. L. Siebert, "Systematic approach to the modelling of measurements for uncertainty evaluation," *Metrologia*, vol. 43, pp. 200–210, 2006.
[14] M. Brandner, T. Thurner, G. Kukutschki, and N. Enzinger, "Optical 2D displacement and strain sensor for creep testing of material samples in transparent fluids," in *Instrumentation and Measurement Technology Conference Proceedings - IMTC2008*, Victoria, Vancouver Island, Canada, May 2008, pp. 1419–1423.
[15] R. I. Hartley and A. Zisserman, *Multiple View Geometry in Computer Vision*, 2^{nd} ed. Cambridge University Press, 2004.