Tutorial 7

Real-Time Volume Graphics

Klaus Engel

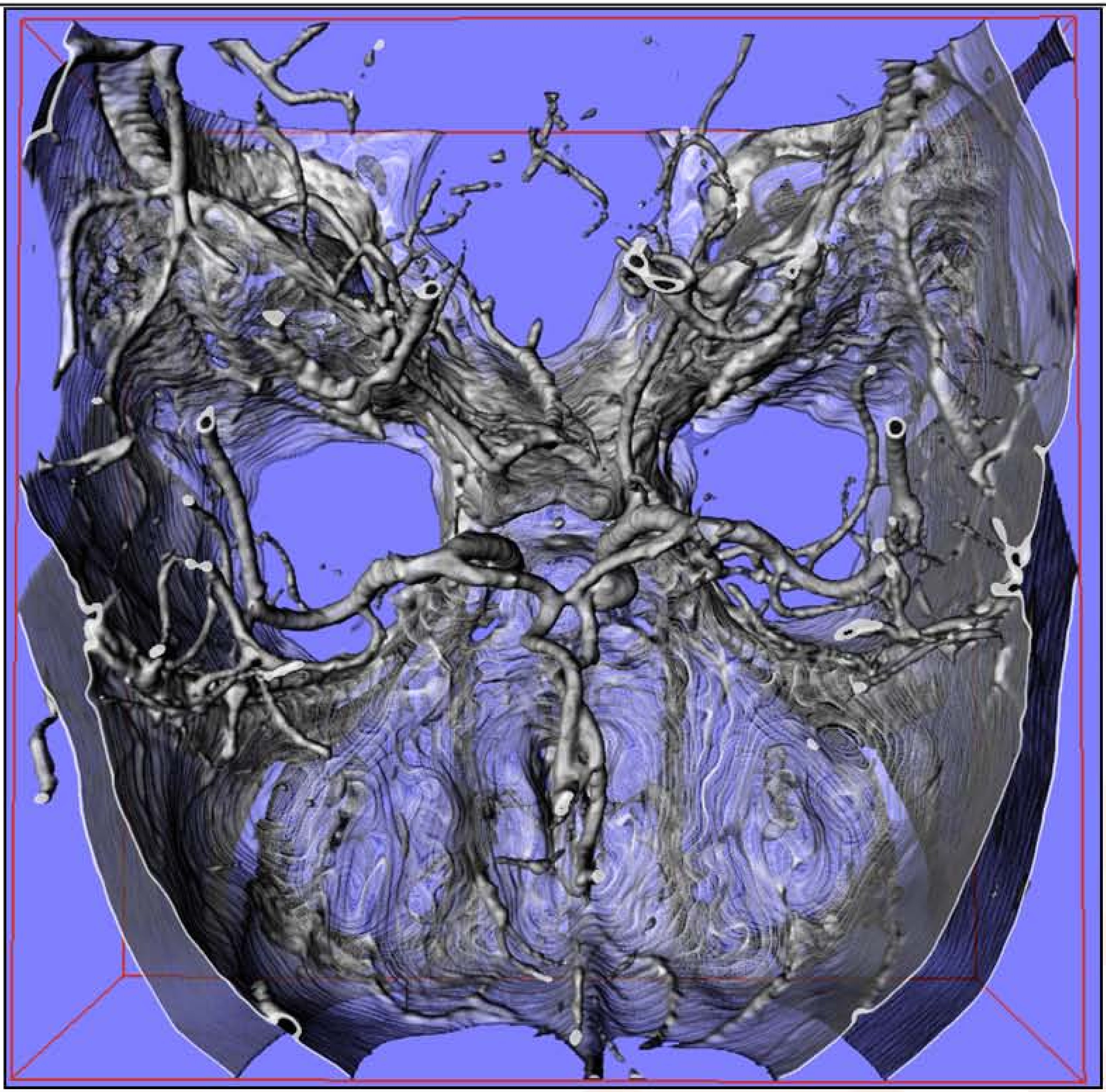
Markus Hadwiger

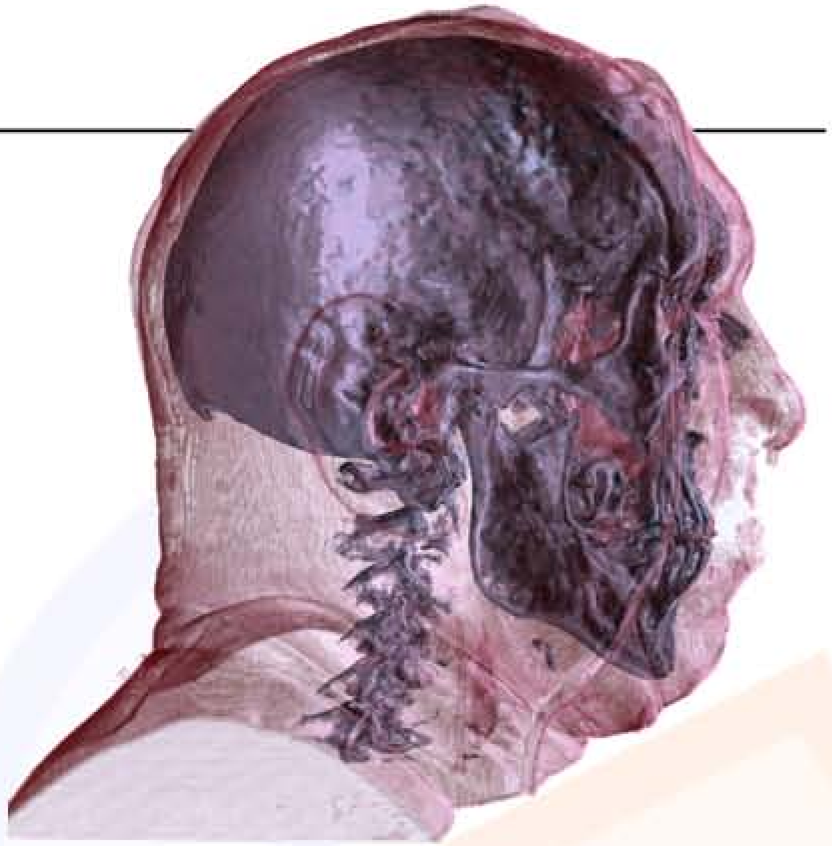
Christof Rezk Salama

Real-Time Volume Graphics

[01] Introduction and Theory

Appliations: Medicine



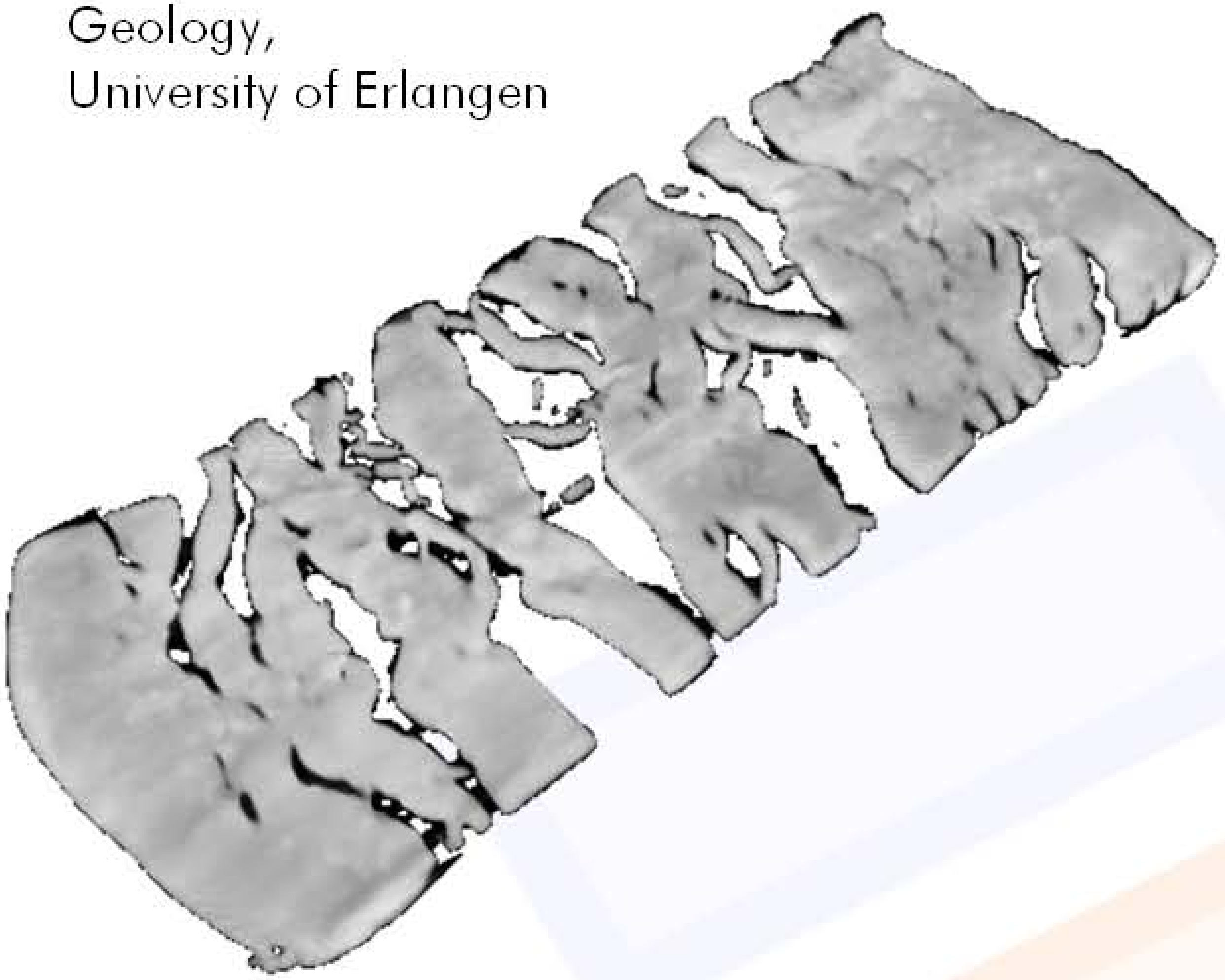


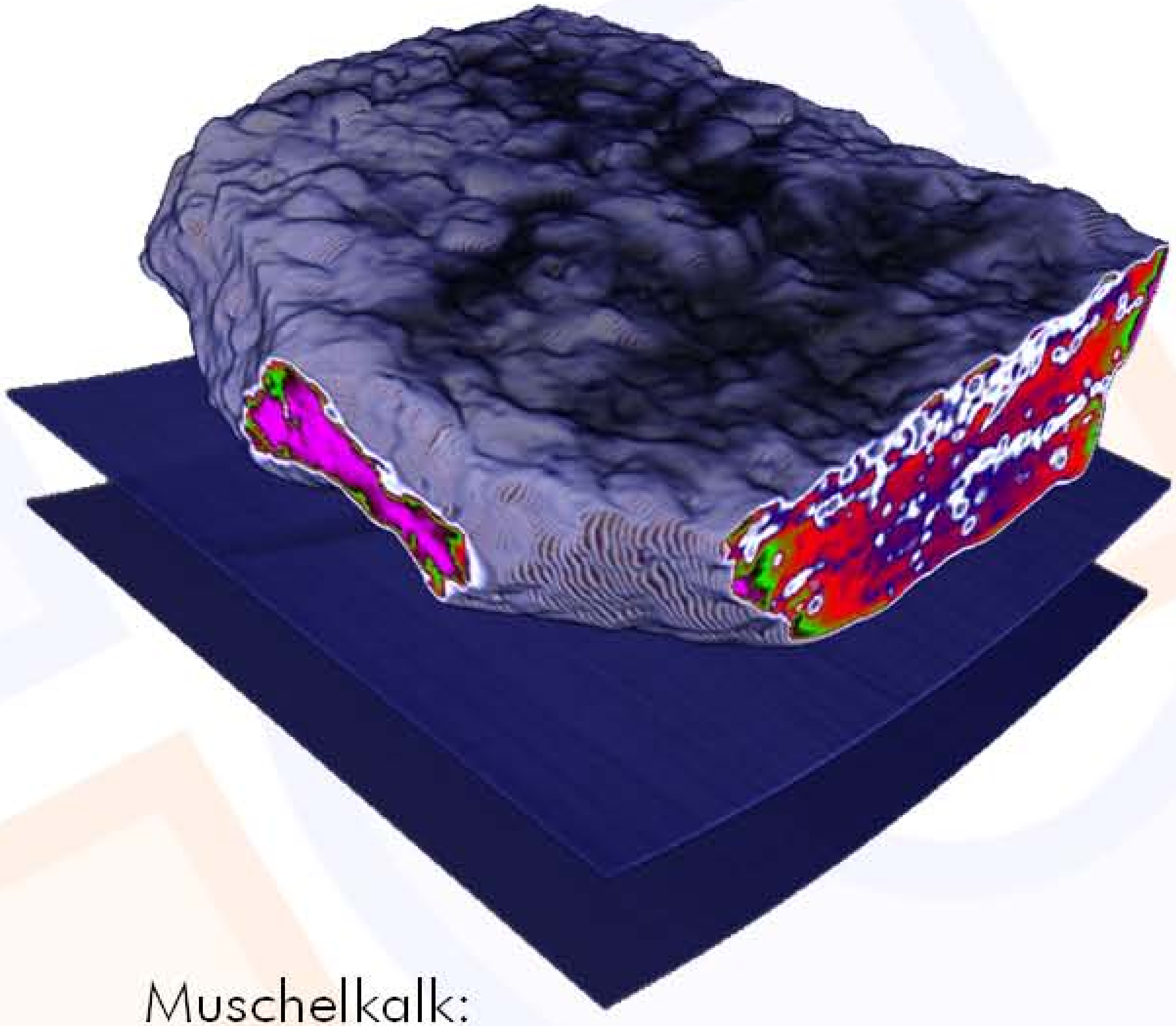
CT Human Head:
Visible Human Project,
US National Library of Medicine,
Maryland,
USA

CT Angiography:
Dept. of Neuroradiology
University of Erlangen,
Germany

Applications: Geology

Deformed Plasticine Model, Applied
Geology,
University of Erlangen





Muschelkalk:
Paläontologie,
Virtual Reality Group,
University of Erlangen

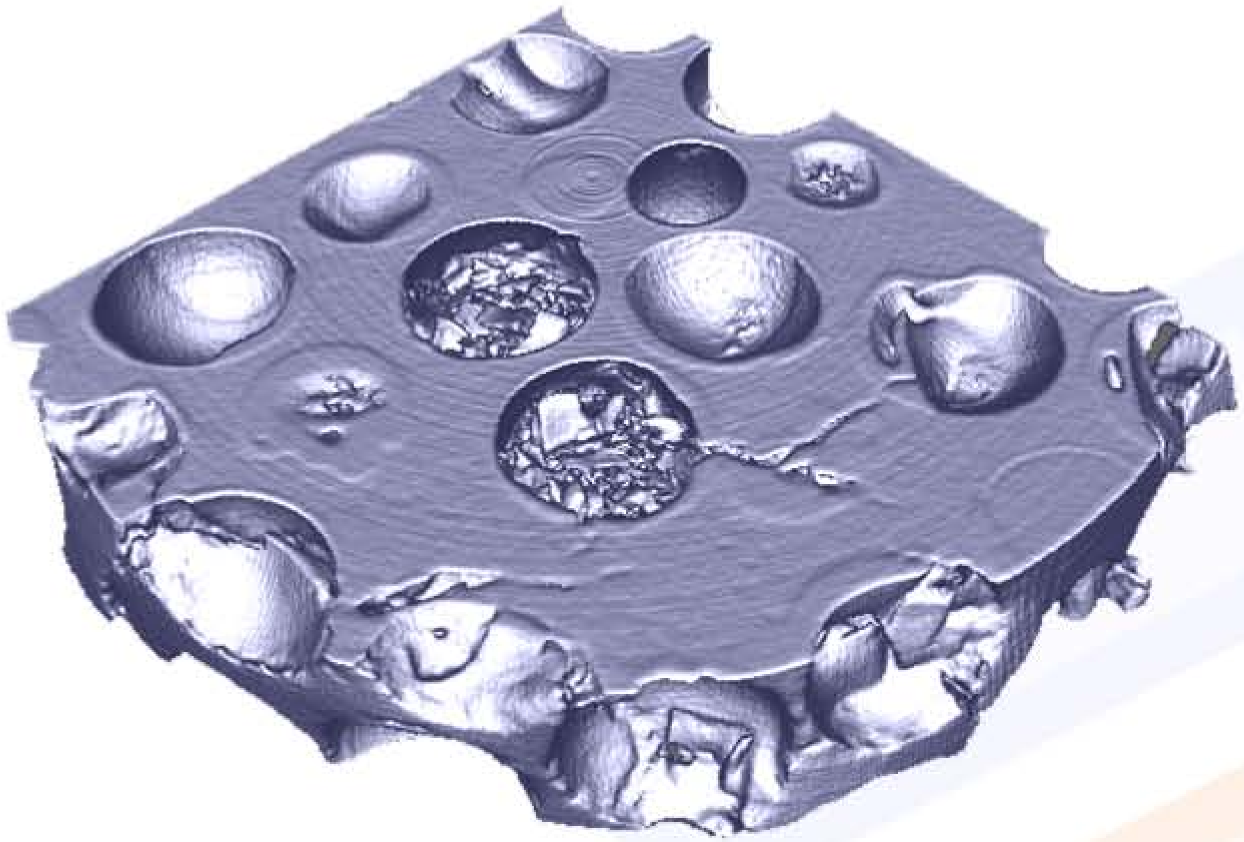
Applications: Archeology

Hellenic Statue of Isis
3rd century B.C.
ARTIS, University of Erlangen-Nuremberg, Germany

Sotades Pygmaios Statue,
5th century B.C
ARTIS, University of Erlangen-Nuremberg, Germany

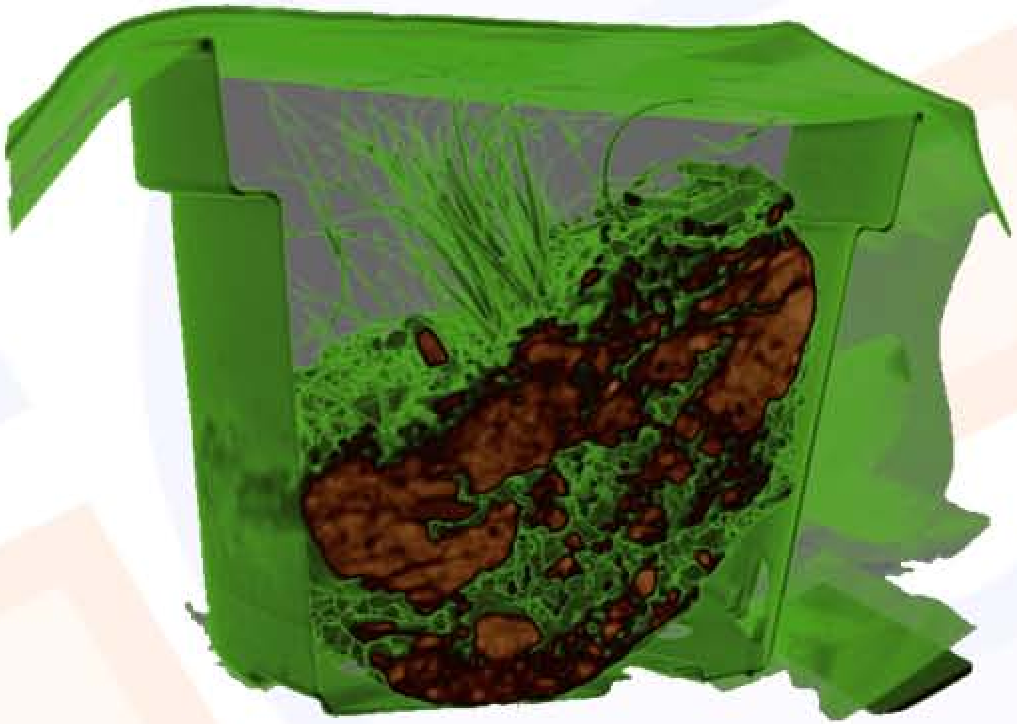
Applications:

Material Science,
Quality Control



Micro CT, Compound Material,
Material Science Department, University of Erlangen

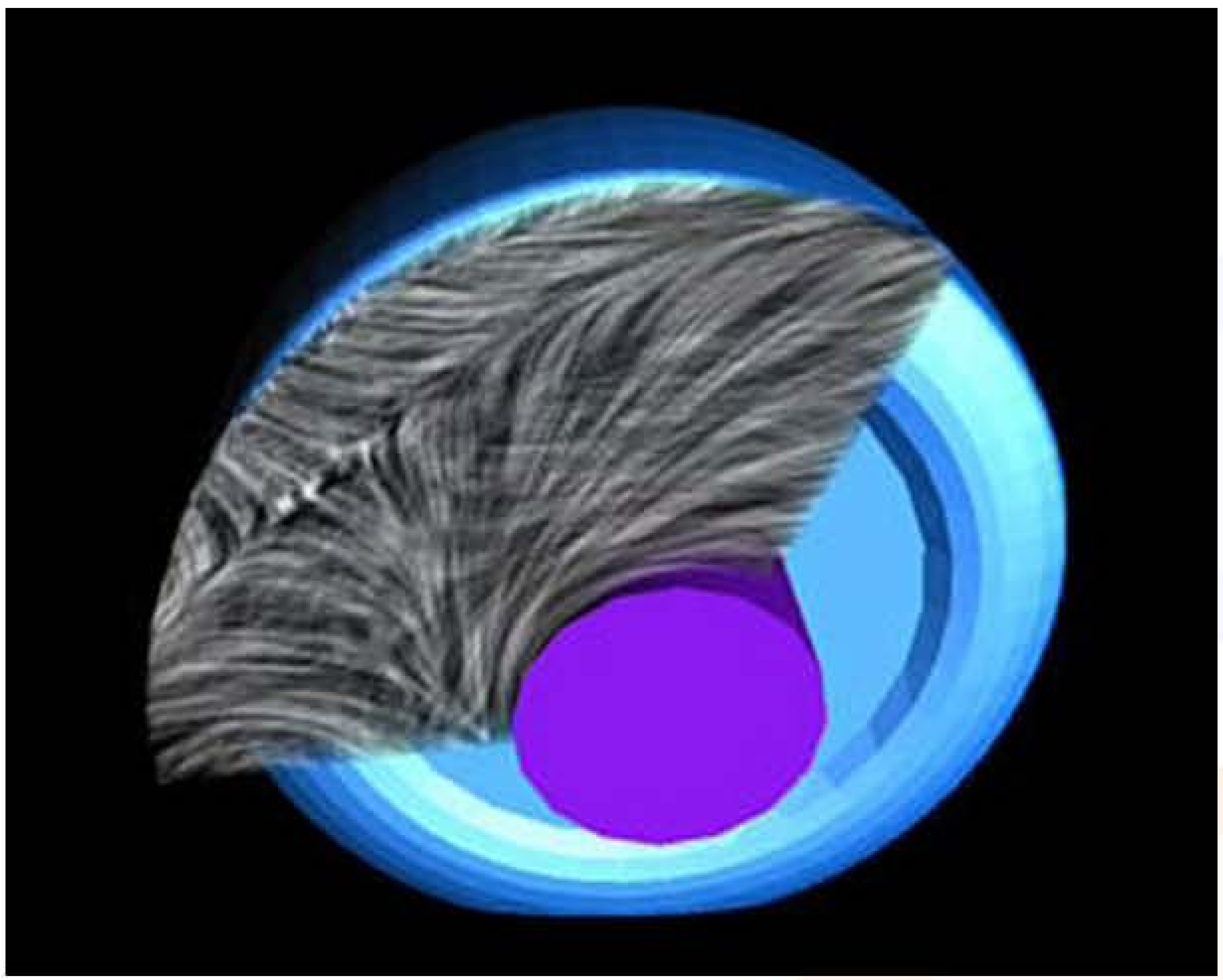
Biology

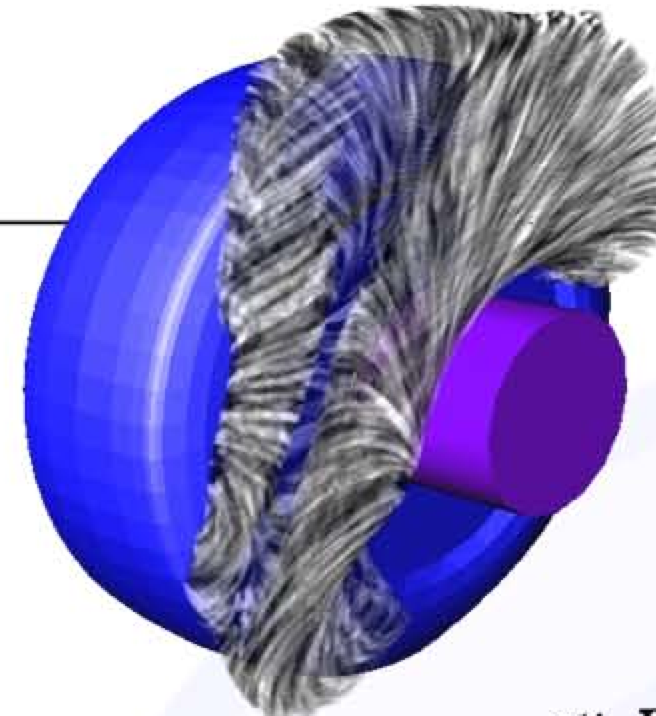


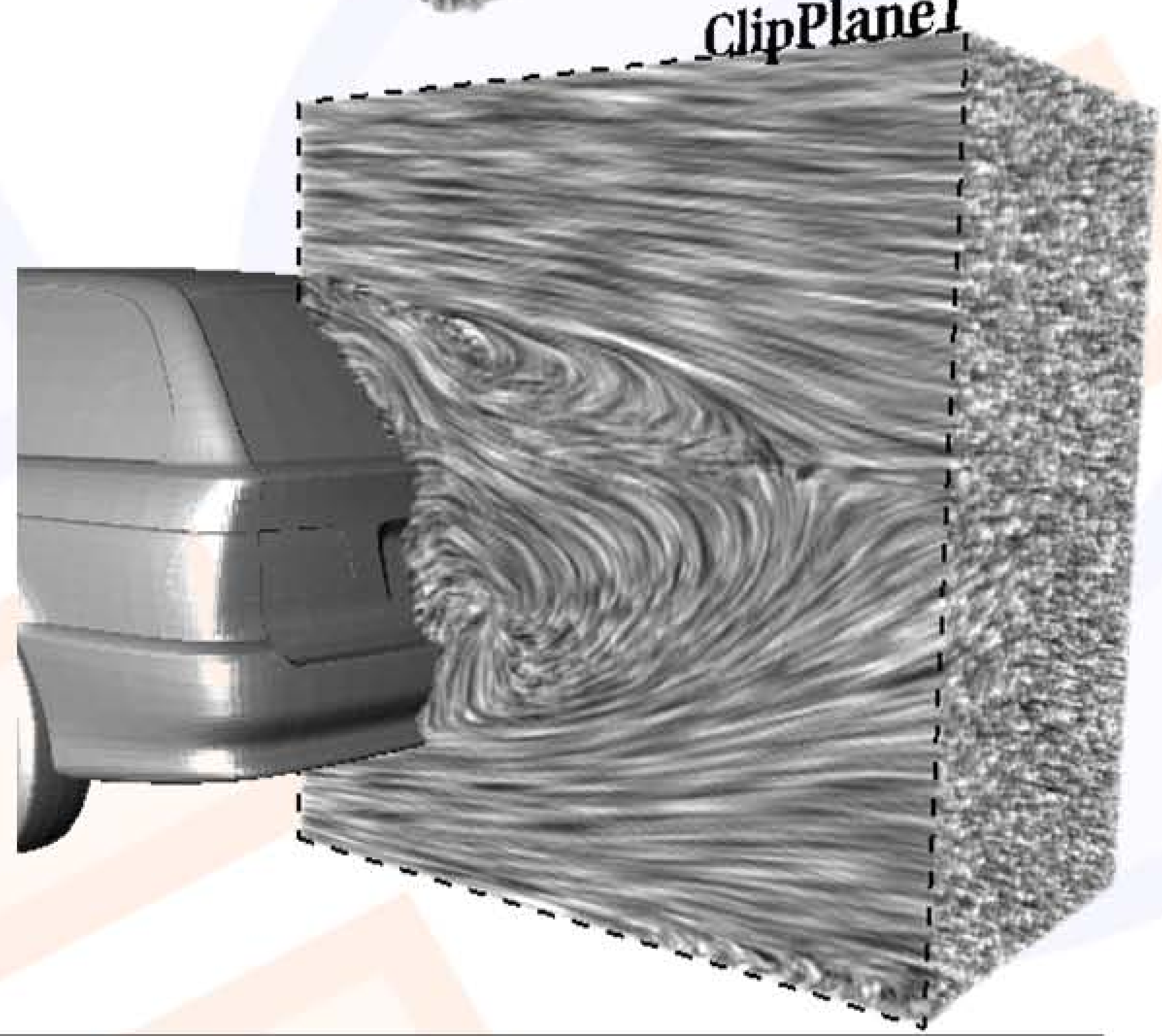
biological sample of the soil, CT,
Virtual Reality Group, University if Erlangen

Applications

Computational Science and Engineering

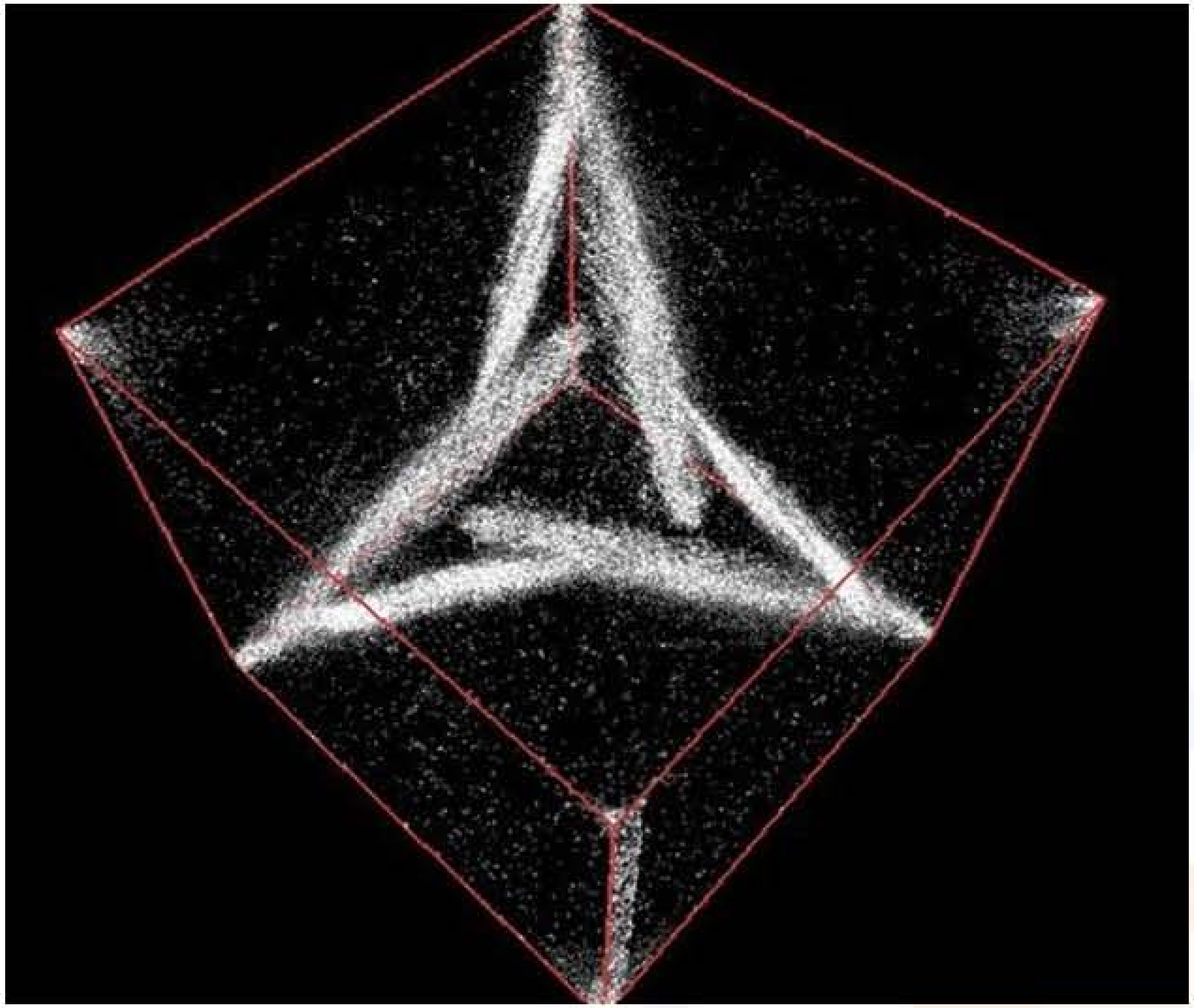


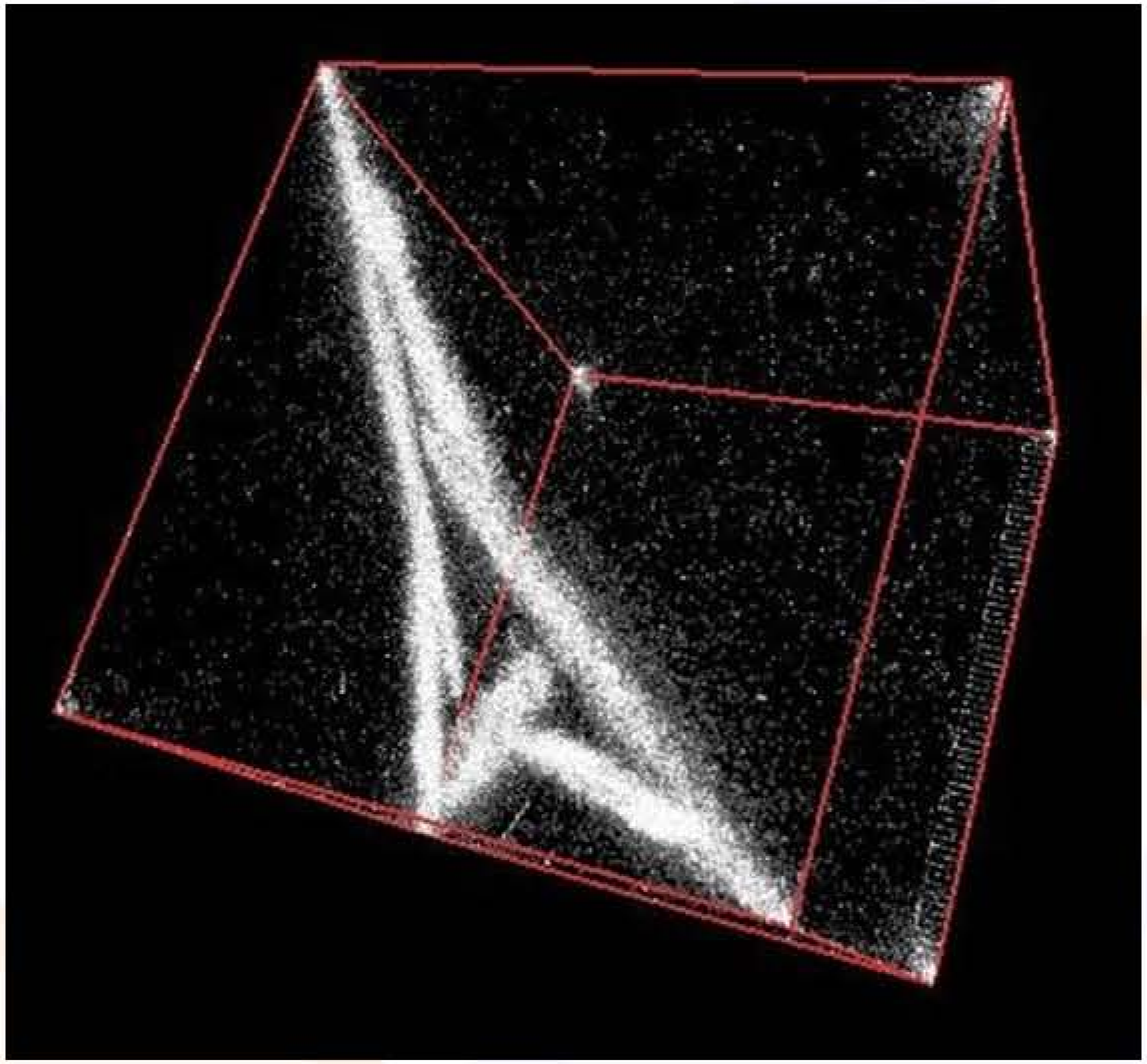




Applications: Computer Science

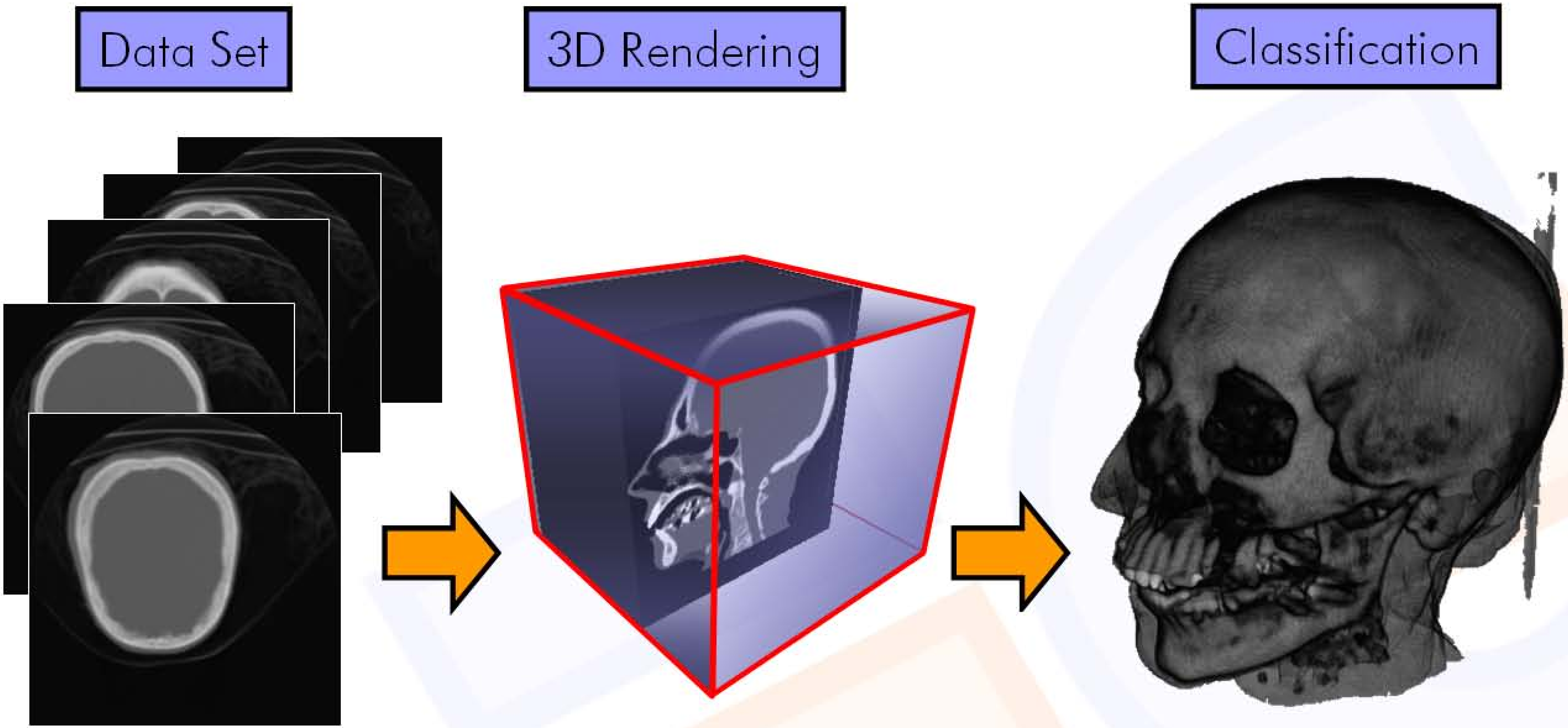
- Visualization of Pseudo Random Numbers





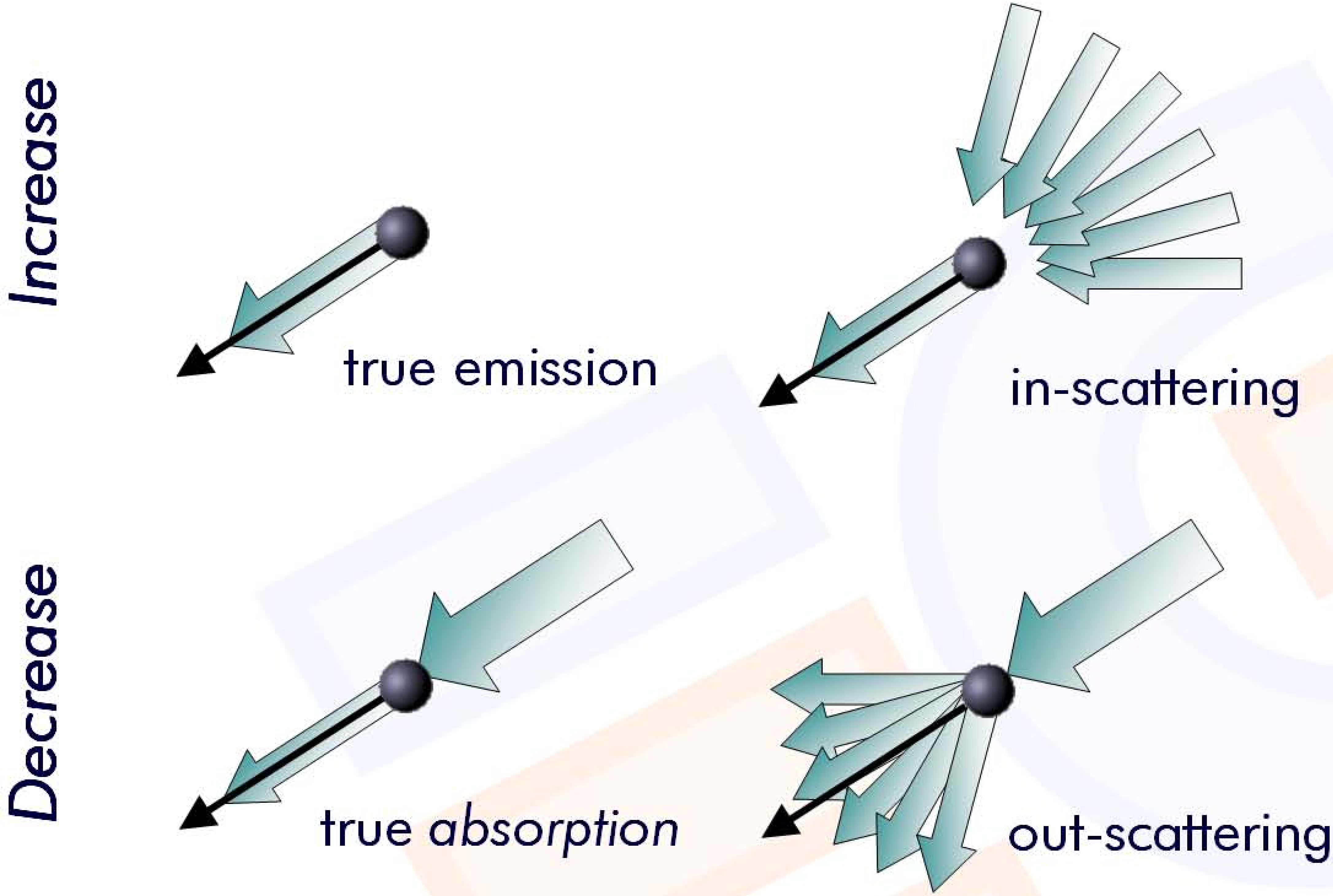
Entropy of Pseudo Random Numbers,
Dan Kaminsky, Doxpara Research, USA,
www.doxpara.com

Outline

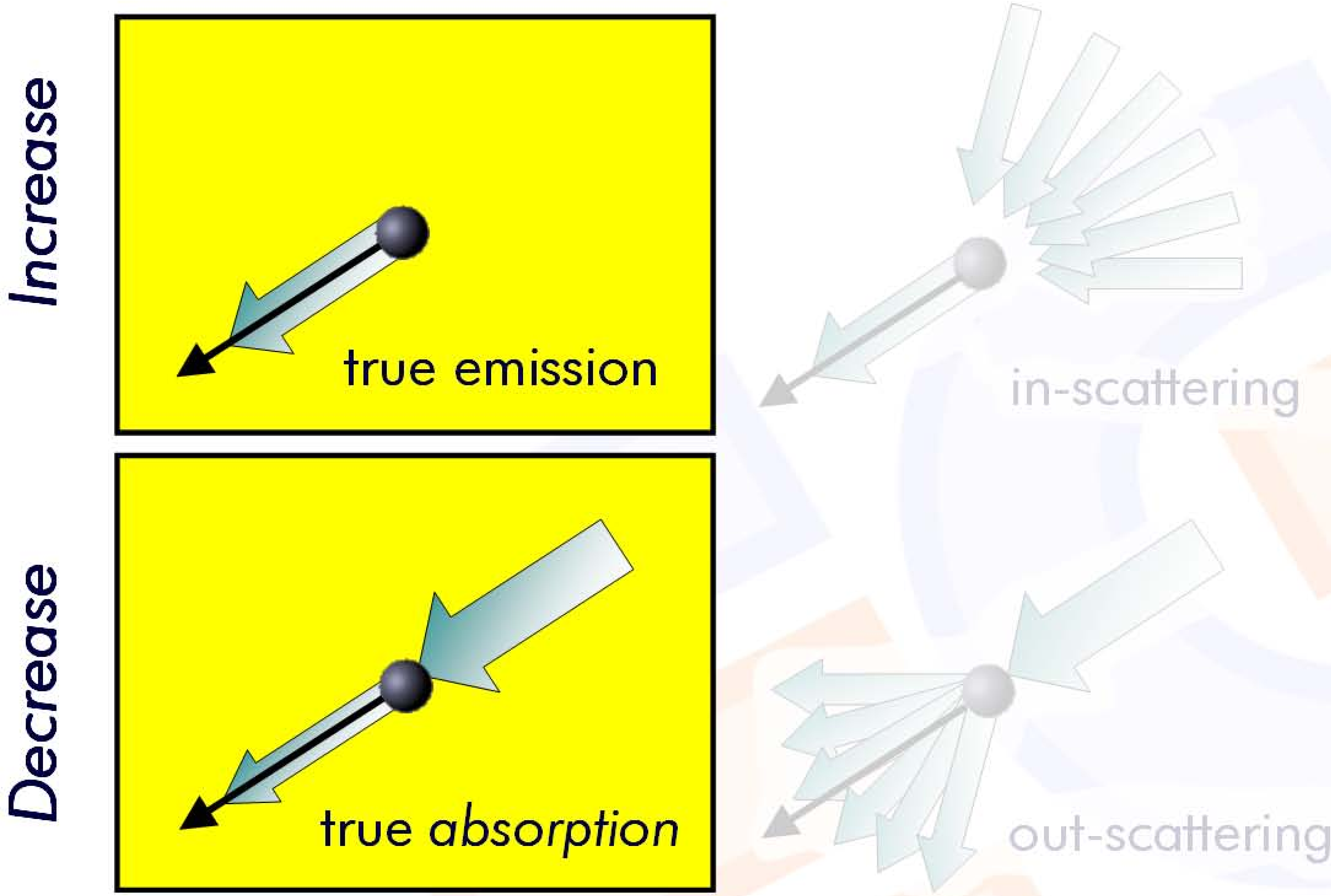


- in real-time on commodity graphics hardware

Physical Model of Radiative Transfer



Physical Model of Radiative Transfer



Ray Integration

How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering

$$I(s) = \boxed{I(s_0)}$$

Ray Integration

How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering

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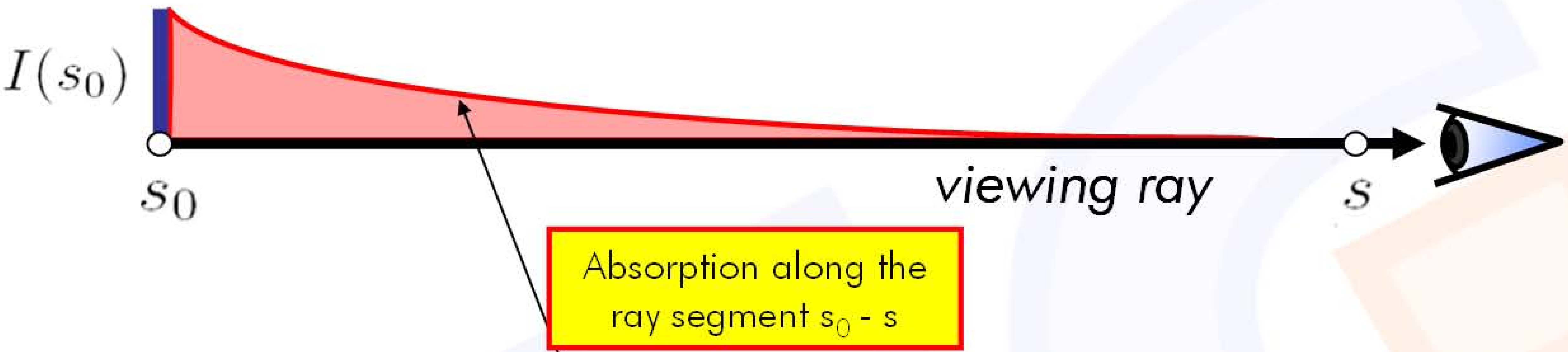
Without absorption all the initial radiant energy would reach the point *s*.

Ray Integration

How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering



$$I(s) = I(s_0)\, e^{-\tau(s_0, s)}$$

Ray Integration

How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering

Extinction τ

Absorption κ

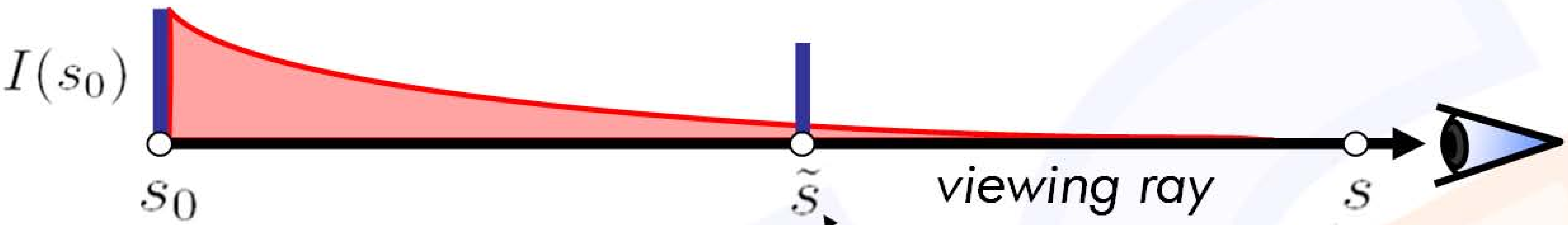
$$\tau(s_1, s_2) = \int_{s_1}^{s_2} \kappa(s)\, ds.$$

$$I(s) = I(s_0)\, e^{-\tau(s_0,s)}$$

Ray Integration

How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering



One point $\tilde{s}$ along the viewing ray emits additional radiant energy.

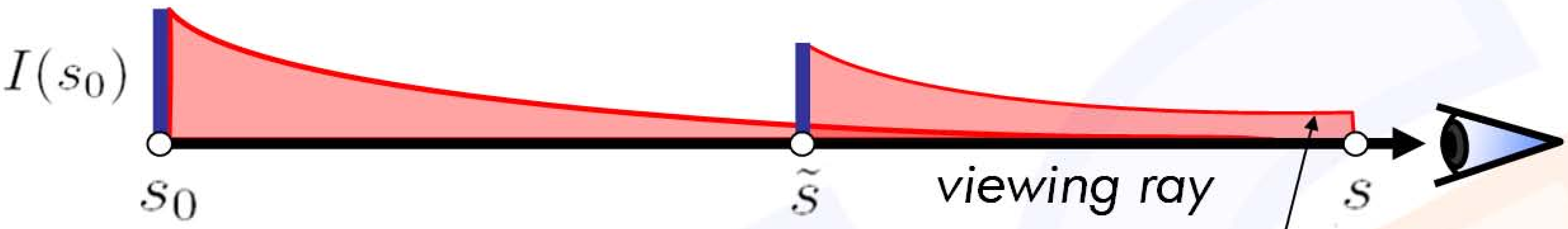
Active emission at point $\tilde{s}$

$$I(s) = I(s_0)\, e^{-\tau(s_0,s)} + q(\tilde{s})$$

Ray Integration

How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering



One point $\tilde{s}$ along the viewing ray emits additional radiant energy.

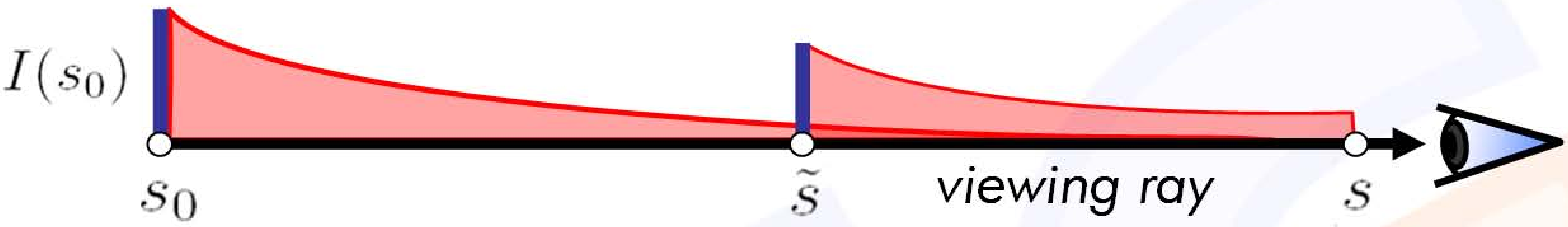
Absorption along the distance s - $\tilde{s}$

$$I(s) = I(s_0)\, e^{-\tau(s_0,s)} + q(\tilde{s})\, e^{-\tau(\tilde{s},s)}$$

Ray Integration

How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering

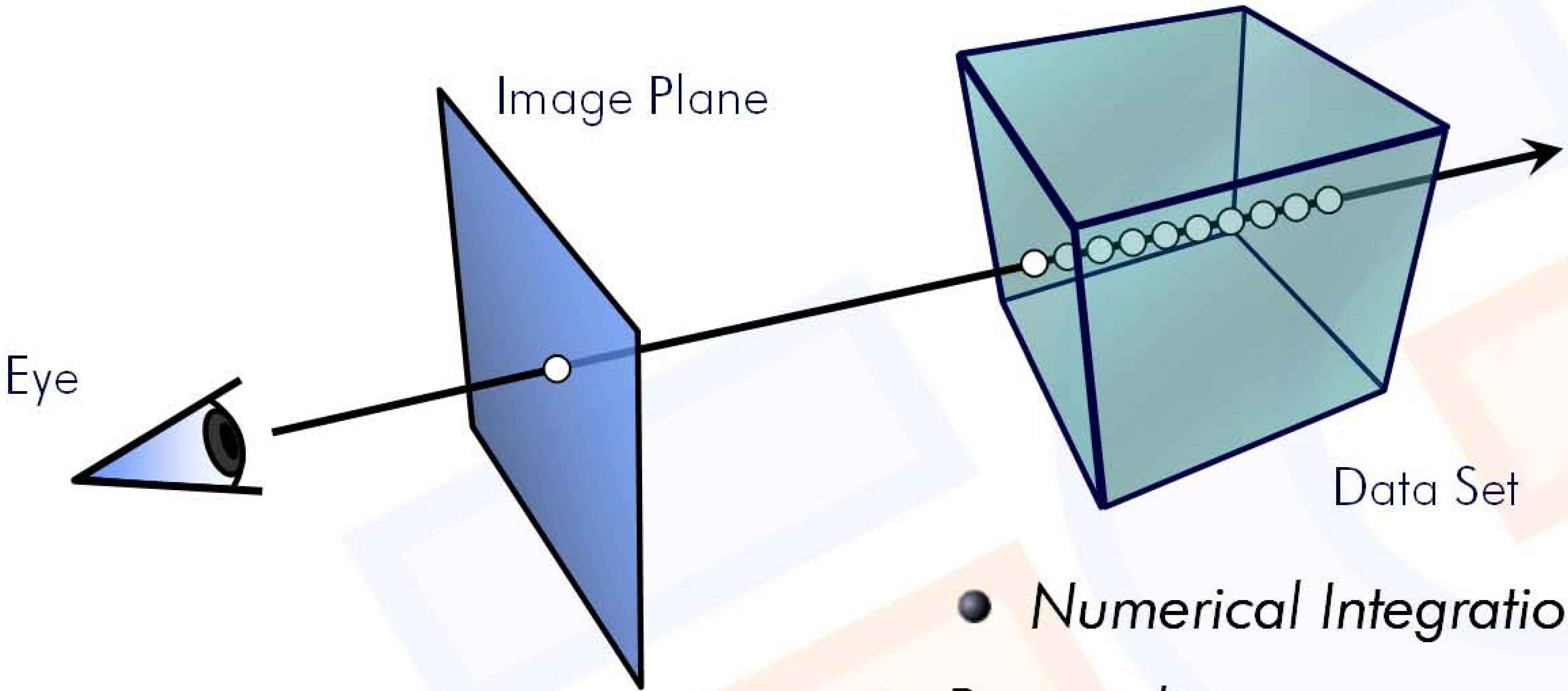


Every point $\tilde{s}$ along the viewing ray emits additional radiant energy

$$I(s) = I(s_0)\, e^{-\tau(s_0, s)} + \int_{s_0}^{s} q(\tilde{s})\, e^{-\tau(\tilde{s}, s)}\, d\tilde{s}$$

Ray Casting

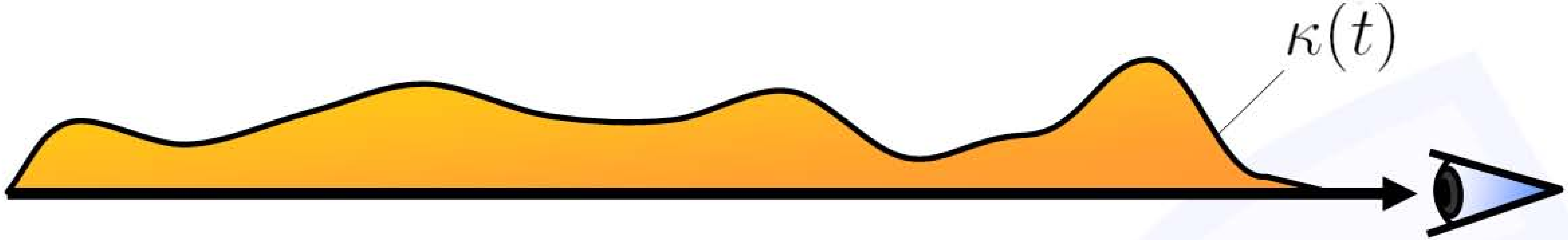
- Software Solution



- *Numerical Integration*
- *Resampling*

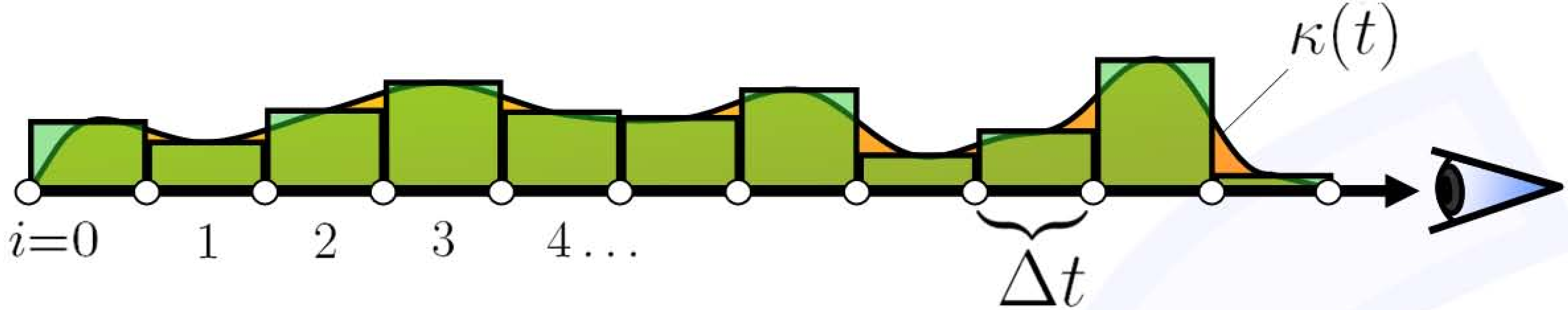
➡ *High Computational Load*

Numerical Solution



Extinction: $$\tau(0,t) = \int_0^t \kappa(\hat{t})\, d\hat{t}$$

Numerical Solution



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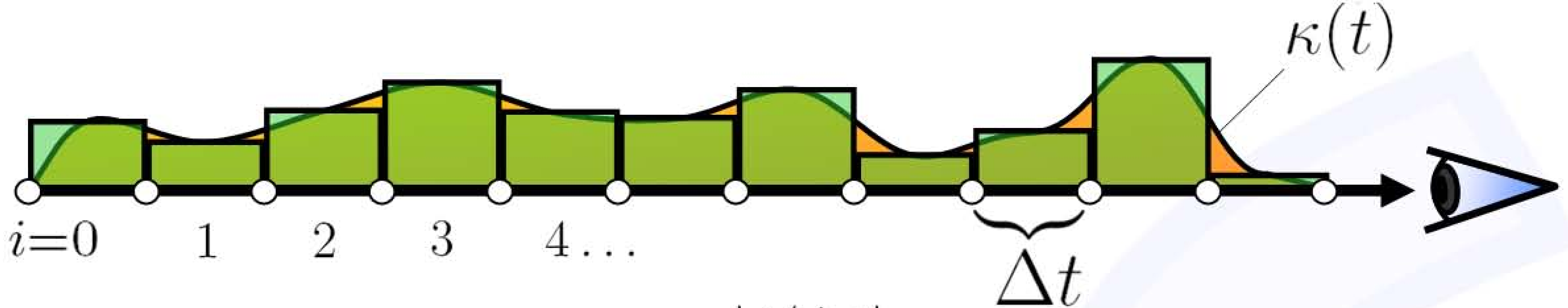
Approximate Integral by Riemann sum:

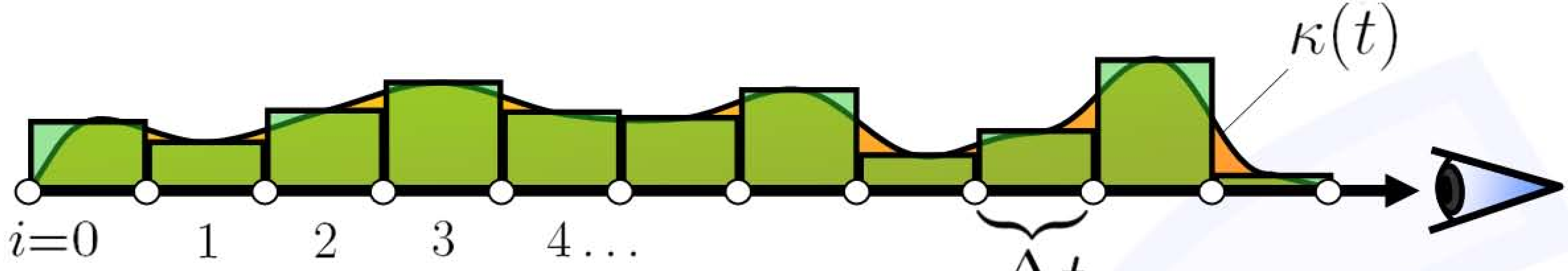
$$\tau(0,t) \;\approx\; \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\,\Delta t$$

Numerical Solution



$$\tau(0,t) \approx \tilde{\tau}(0,t) = \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\, \Delta t$$

Numerical Solution



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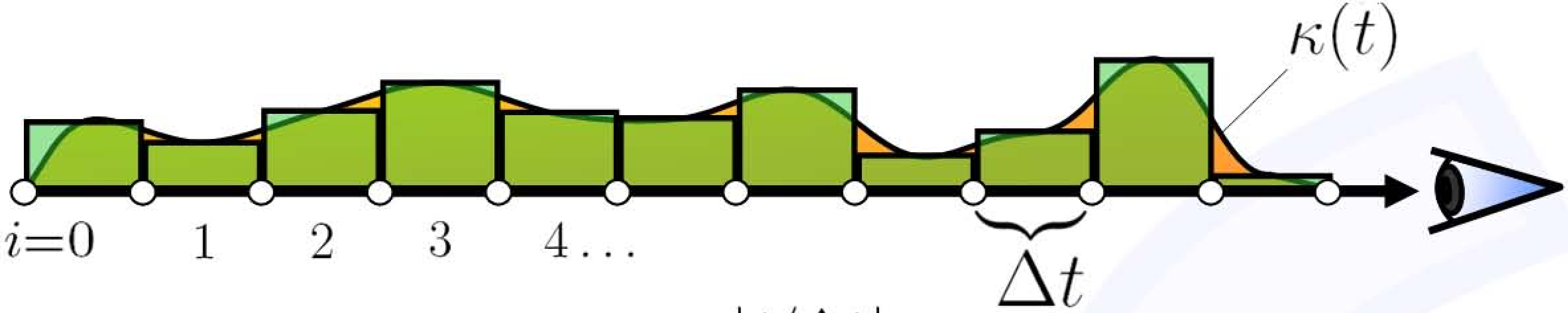
$$e^{-\tilde{\tau}(0,t)} \;=\; e^{-\sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\, \Delta t}$$

Numerical Solution



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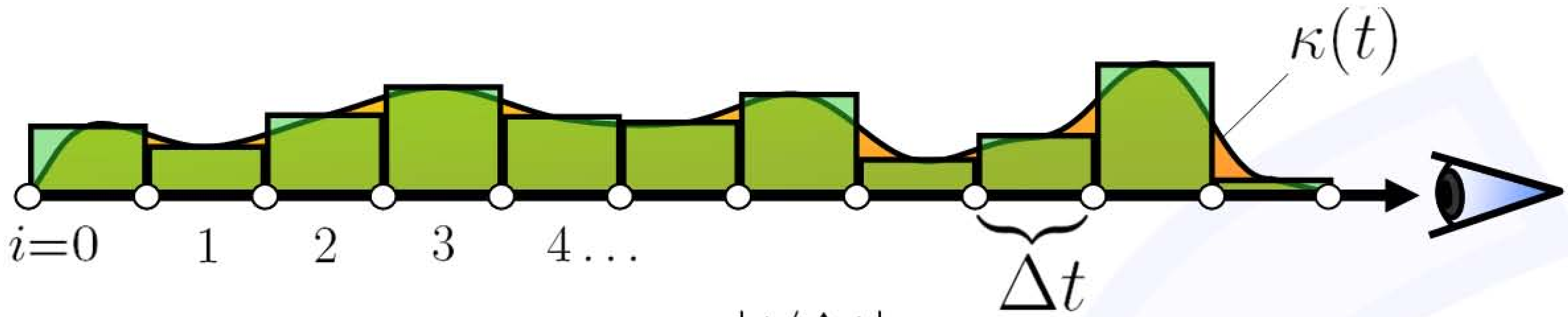

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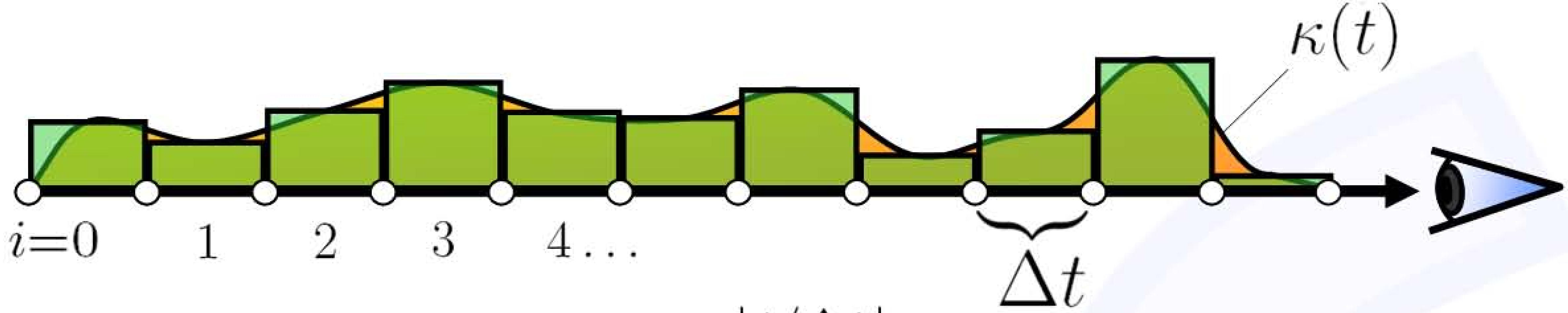
Now we introduce opacity:

$$A_i = 1 - e^{-\kappa(i \cdot \Delta t)\,\Delta t}$$

Numerical Solution



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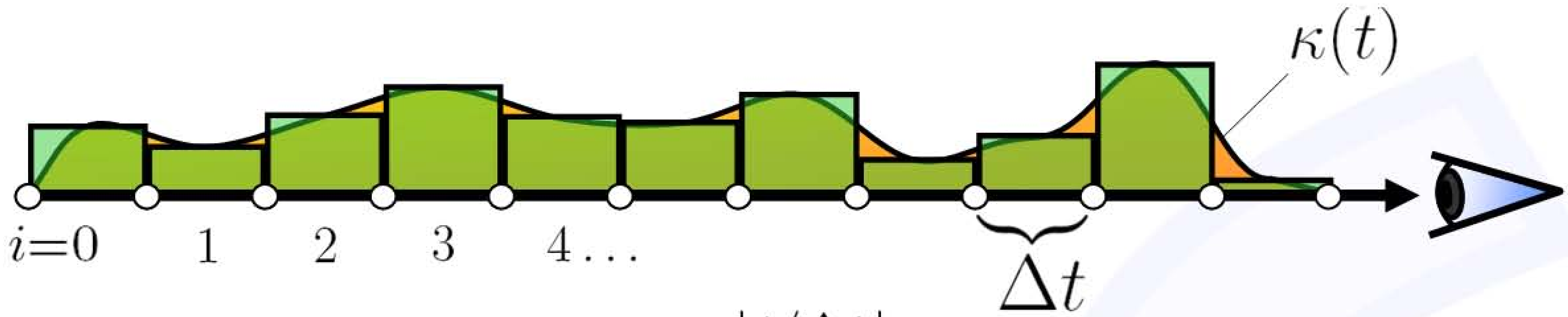
Numerical Solution



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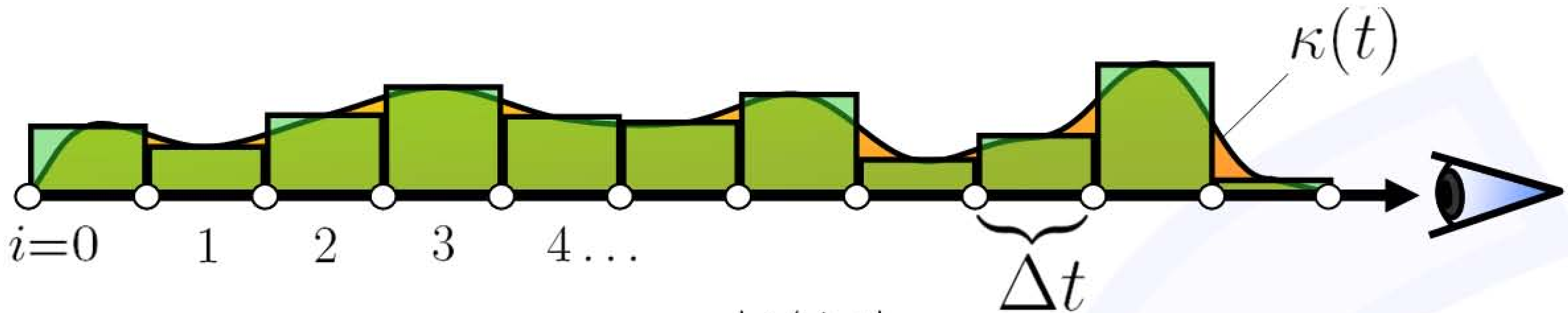

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$$e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} (1 - A_i)$$

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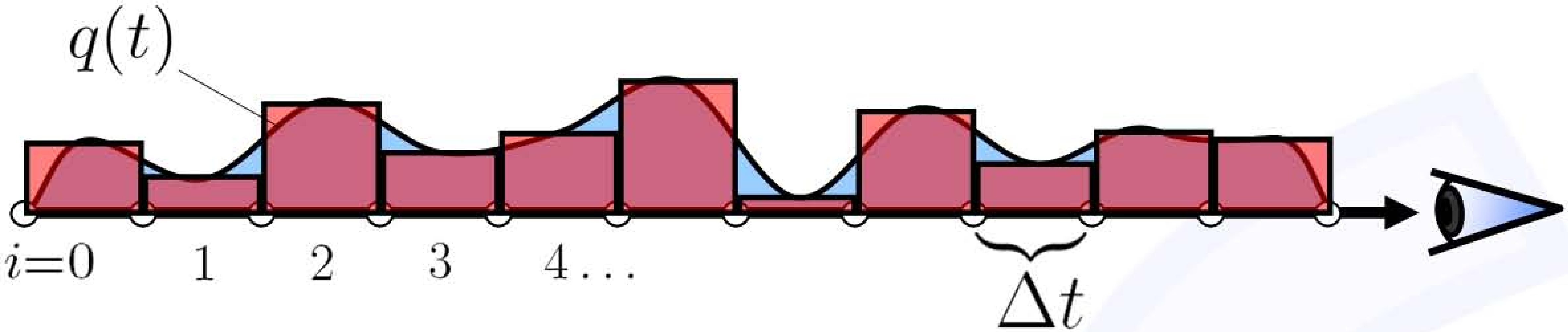
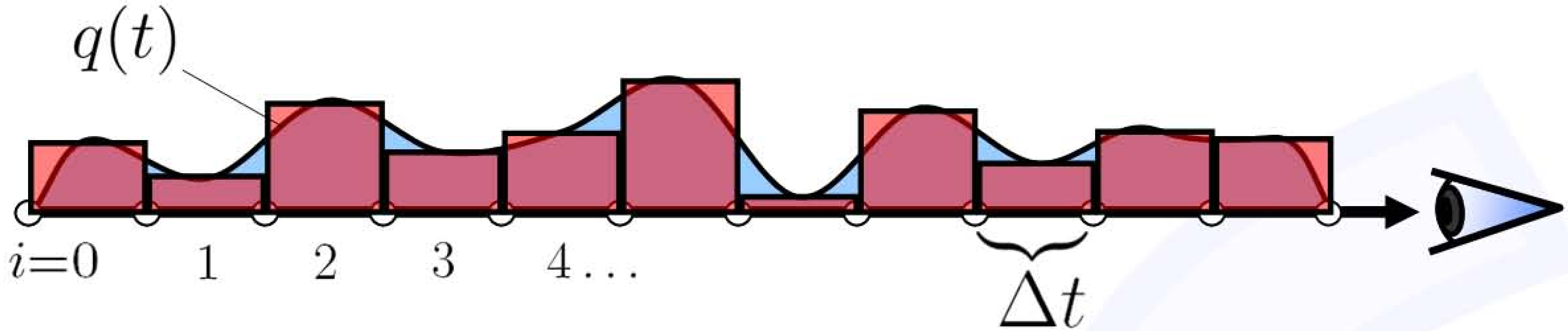
Numerical Solution



$$e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} (1 - A_i)$$

$$q(t) \approx C_i = c(i \cdot \Delta t)\,\Delta t$$

Numerical Solution



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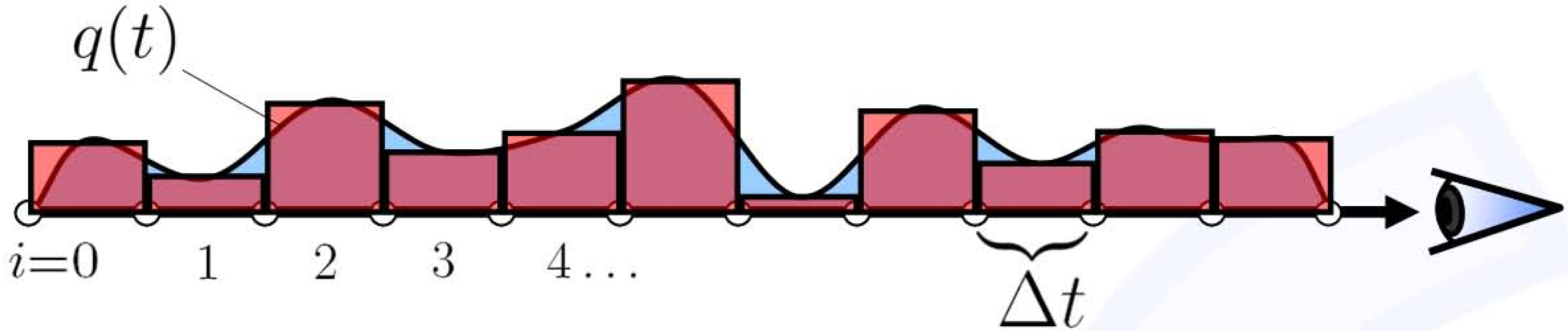
$$\tilde{C} = \sum_{i=0}^{\lfloor T/\Delta t \rfloor} C_i \, e^{-\tilde{\tau}(0,t)}$$

Numerical Solution



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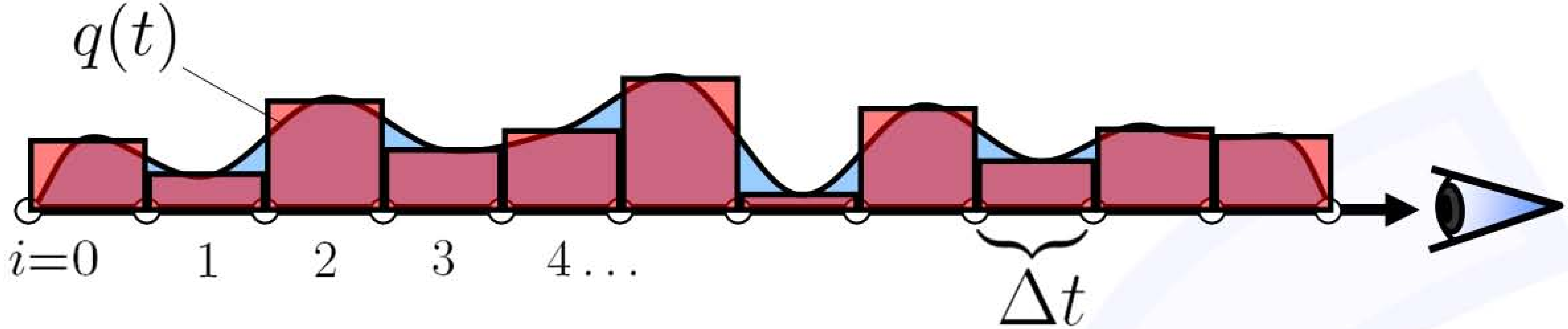

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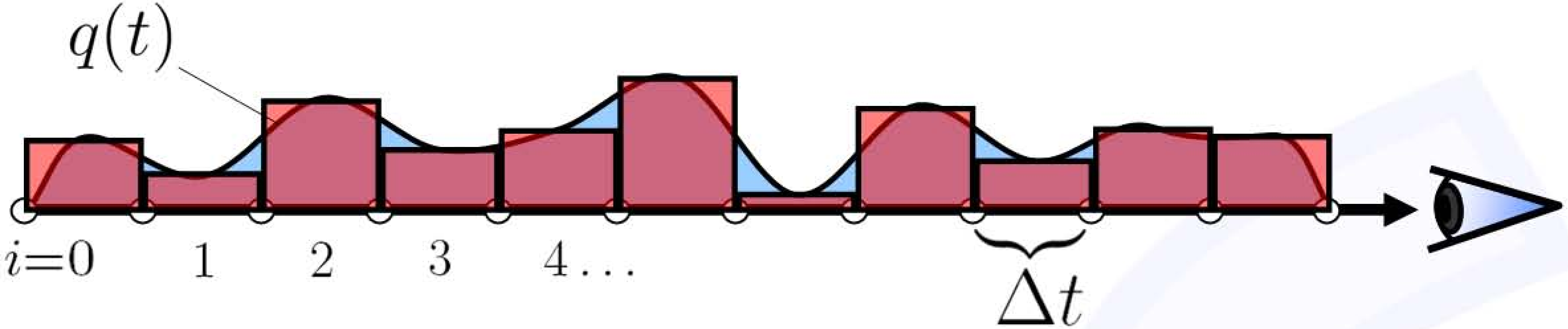
$$\tilde{C} = \sum_{i=0}^{\lfloor T/\Delta t \rfloor} C_i \prod_{j=0}^{i-1} (1 - A_j)$$

Numerical Solution



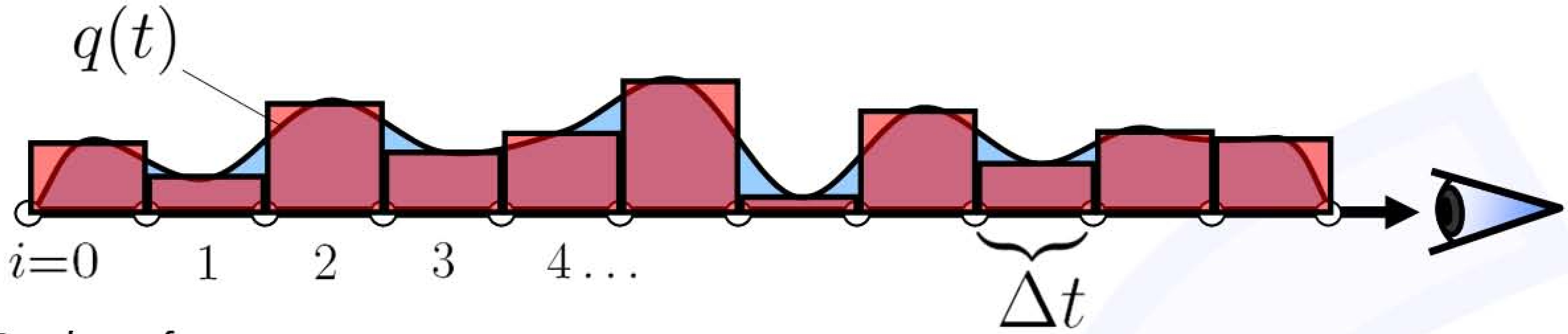
$$\tilde{C} = \sum_{i=0}^{\lfloor T/\Delta t \rfloor} C_i \prod_{j=0}^{i-1} (1 - A_j)$$

can be computed recursively

$$C'_i = C_i + (1 - A_i) C'_{i-1}$$

- C'_i: Radiant energy observed at position *i*
- C_i: Radiant energy emitted at position *i*
- A_i: Absorption at position *i*
- C'_{i-1}: Radiant energy observed at position *i*–1

Numerical Solution



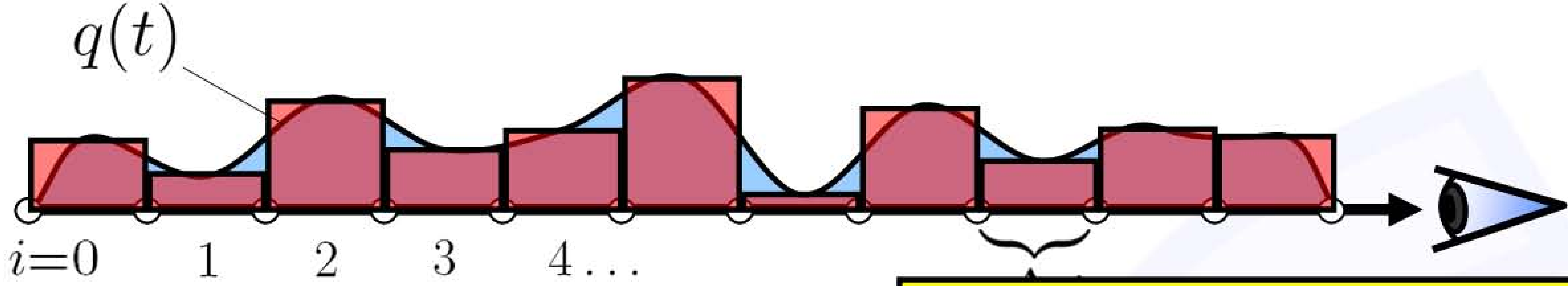
Back-to-front compositing

$$C'_i = C_i + (1 - A_i)C'_{i-1}$$

Front-to-back compositing

$$C'_i = C'_{i+1} + (1 - A'_{i+1})C_i$$
$$A'_i = A'_{i+1} + (1 - A'_{i+1})A_i$$

Numerical Solution



Back-to-front compositing

$$C'_i = C_i + (1$$

Front-to-back compositing

$$C'_i = C'_{i+1} + (1 - A'_{i+1}) C_i$$

$$A'_i = A'_{i+1} + (1 - A'_{i+1}) A_i$$

Early Ray Termination:

Stop the calculation when

$$A'_i \approx 1$$

Summary

- Emission Absorption Model

true emission true absorption

$$I(s) \;=\; I(s_0)\, e^{-\tau(s_0,s)} + \int_{s_0}^{s} q(\tilde{s})\, e^{-\tau(\tilde{s},s)}\, \mathrm{d}\tilde{s}$$

- Numerical Solutions

Back-to-front iteration

$$C'_i \;=\; C_i + (1 - A_i) C'_{i-1}$$

Front-to-back iteration

$$\begin{aligned} C'_i &= C'_{i+1} + (1 - A'_{i+1}) C_i \\ A'_i &= A'_{i+1} + (1 - A'_{i+1}) A_i \end{aligned}$$