Computer Graphics II
5: Subdivision Surfaces

Computer Graphics and
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Structure of this Chapter

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- *Subsection 1: Basic approach discussed for curves*
- *Subsection 2: Extension to surfaces*

Motivation

Goal: *Combing the advantages of free-form-surfaces and polygon meshes:*

- *Efficient storage and simple control (free-form-surfaces)*
- *Random complex forms without continuity conditions (meshes)*

Approach: *Recursive refinement (subdivision) of a* coarse polygon mesh $\mathcal{P}^0$
$\mathcal{P}^0 \longrightarrow \mathcal{P}^1 \longrightarrow \cdots \mathcal{P}^\infty$ *such that*

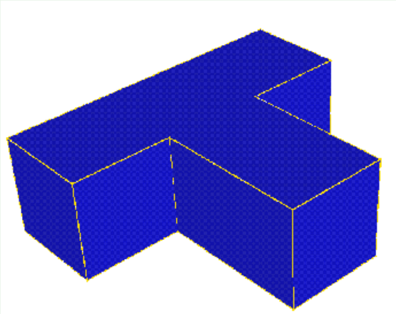
1. *the refinement process converges to a limit surface* $\mathcal{P}^\infty$
2. *the limit surface* ($\longrightarrow \infty$) *is smooth* (C^1 or C^2)

Different kinds of subdivision procedures: *Approximating* or *interpolating techniques*

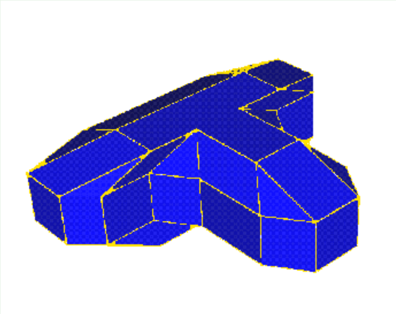
Example (Geri's Game, Pixar 1997)

- Geri's Game was the first movie using subdivision surfaces throughout

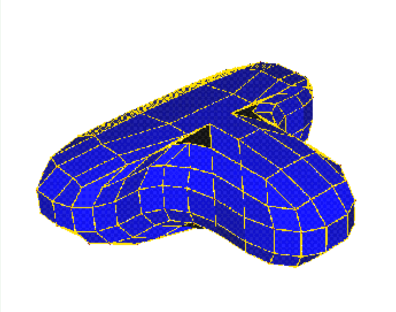
Example



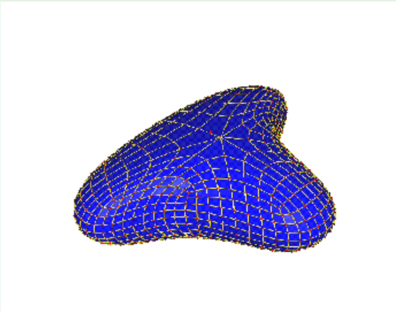
Initial mesh



One subdivision step



Two subdivision steps



Three subdivision steps

Approach (Subdivision Curves Chaikin '74)

Given: *A sequence of* $k+1$ *points* $\{\mathbf{P}_0^0, \ldots, \mathbf{P}_k^0\}$

Subdivision Rule: *On each edge* two *new points are defined:*

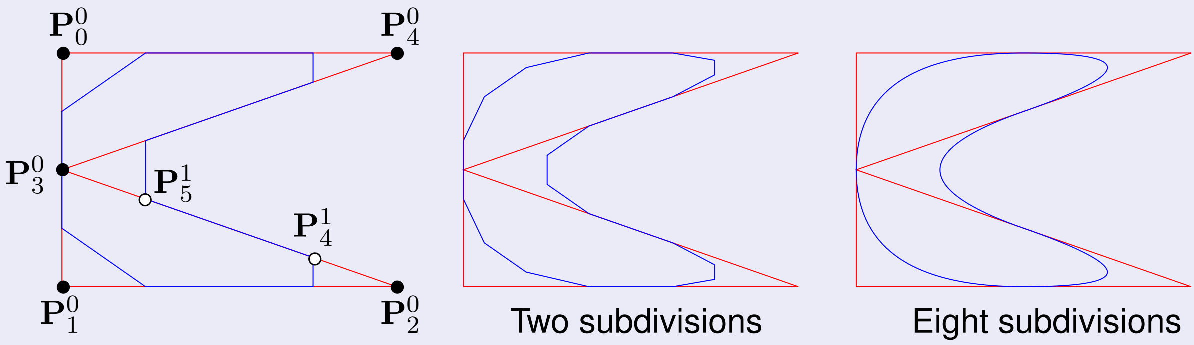
$$\mathbf{P}_{2j}^{i+1} = \frac{3}{4}\mathbf{P}_j^i + \frac{1}{4}\mathbf{P}_{j+1}^i \qquad \mathbf{P}_{2j+1}^{i+1} = \frac{1}{4}\mathbf{P}_j^i + \frac{3}{4}\mathbf{P}_{j+1}^i$$

The result is an approximating scheme.

Property: *This scheme corresponds to quadratic b-splines, whereby the convex hull property holds*

Approach (Subdivision Curves Chaikin '74 (cont'))

Example Curve (closed):



Reminder: *Knot insertion in quadratic b-splines*

- *Given: Uniform knot vector* $T = 0, 1, 2, \ldots, m+3$
- *Insertion of knots at* $i + 0.5, i = 0, \ldots, m+2$ *delivers precisely the Chaikin-scheme*

Approach (4-Point Subdivision-Scheme)

Given: *Sequence of* $k+1$ *points* $\{\mathbf{P}_0^0, \ldots, \mathbf{P}_k^0\}$

Subdivision Rule: *One additional new point on each edge:*

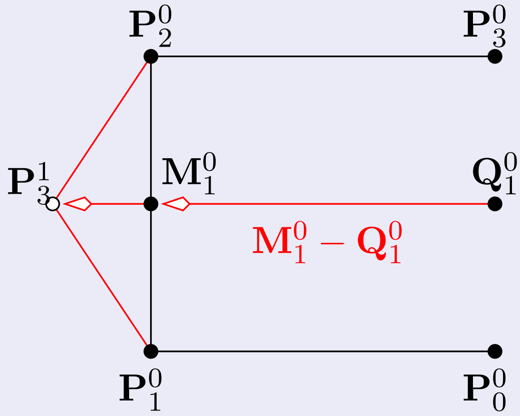
$$\mathbf{P}_{2j}^{i+1} = \mathbf{P}_j^i, \qquad \mathbf{P}_{2j+1}^{i+1} = \left(\frac{1}{2} + w\right)\left(\mathbf{P}_j^i + \mathbf{P}_{j+1}^i\right) - w\left(\mathbf{P}_{j-1}^i + \mathbf{P}_{j+2}^i\right)$$

Explanation *of the construction.*

Edge center $\mathbf{M}_j^i = \frac{1}{2}(\mathbf{P}_j^i + \mathbf{P}_{j+1}^i)$

Center parent and successor $\mathbf{Q}_j^i = \frac{1}{2}(\mathbf{P}_{j-1}^i + \mathbf{P}_{j+2}^i)$

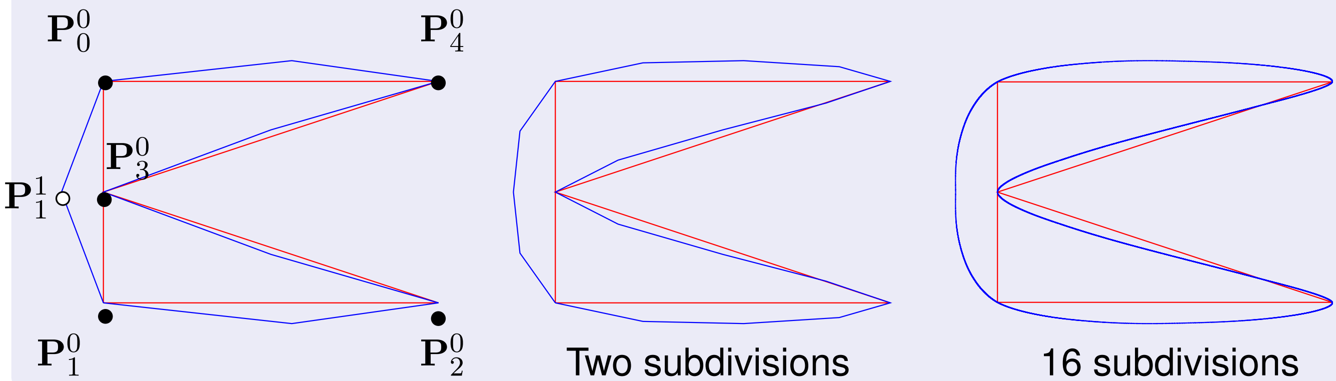
then is $\mathbf{P}_{2j+1}^{i+1} = \mathbf{M}_j^i + 2w(\mathbf{M}_j^i - \mathbf{Q}_j^i)$



Approach (4-Point Subdivision Scheme (Cont.))

Properties: *Original points remain intact (*interpolating scheme*)*
From $w = \frac{1}{16}$ *results a visually smooth curve.*

Example curve (closed): $w = \frac{1}{16}$



Approach (Doo-Sabin Subdivision Surfaces)

Given: *2-manifold polygon mesh* $\mathcal{P}^0$ *with the vertices* $\mathbf{P}^0$

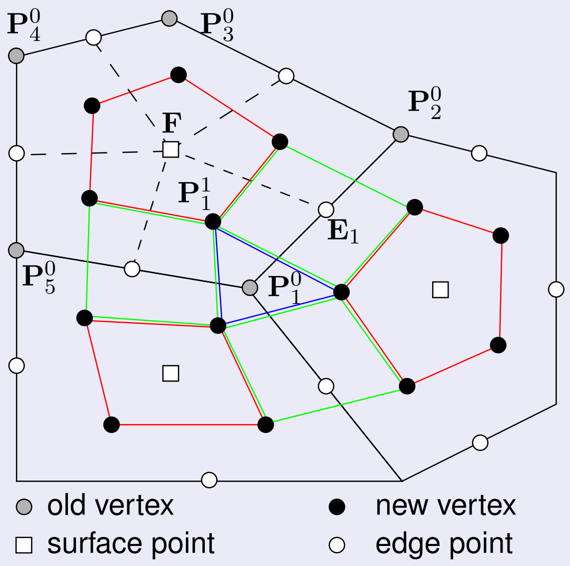
Subdivision Rule: *Consider polygon* $(\mathbf{P}_1^0, \ldots, \mathbf{P}_k^0)$ *of* $\mathcal{P}^0$*. construction:*

Surface point: $\mathbf{F} = \frac{1}{k}\sum_{i=1}^{k} \mathbf{P}_i^0$

Edge point: $\mathbf{E}_i = \frac{1}{2}(\mathbf{P}_i^0 + \mathbf{P}_{i+1}^0),$

$i = 1, \ldots, k, \ \mathbf{P}_{k+1}^0 := \mathbf{P}_1^0$

Vertex: $\mathbf{P}_i^1 = \frac{1}{4}(\mathbf{F} + \mathbf{E}_{i-1} + \mathbf{E}_i + \mathbf{P}_i^0)$



In each step we get

$\mathcal{P}^i$	$\mathcal{P}^{i+1}$
k-sided polygon	k-sided polygon
k-valent vertex	k-sided polygon
edge	4-sided polygon

Property

Subdivision Mask: *Matrix representation of a subdivision step:*

$$\begin{pmatrix} \mathbf{P}_1^1 \\ \vdots \\ \mathbf{P}_k^1 \end{pmatrix} = \frac{1}{8k} \underbrace{\begin{pmatrix} 4k+2 & k+2 & 2 & 2 & 2 & \cdots & 2 & k+2 \\ k+2 & 4k+2 & k+2 & 2 & 2 & \cdots & 2 & 2 \\ 2 & k+2 & 4k+2 & k+2 & 2 & \cdots & 2 & 2 \\ \vdots & & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 2 & 2 & \cdots & 2 & k+2 & 4k+2 & k+2 & 2 \\ 2 & 2 & 2 & \cdots & 2 & k+2 & 4k+2 & k+2 \\ k+2 & 2 & 2 & 2 & \cdots & 2 & k+2 & 4k+2 \end{pmatrix}}_{=:S} \cdot \begin{pmatrix} \mathbf{P}_1^0 \\ \vdots \\ \mathbf{P}_k^0 \end{pmatrix}$$

Doo-Sabin surfaces *generalizes* quadratic *b-spline surfaces.*

k-valent vertex/k-sided surface, $k \neq 4$ *delivers for* $\mathcal{P} = \mathcal{P}^\infty$ *extraordinary points*

- *Outside of these points:* $\mathcal{P}$ *is a local* C^1*-steady, quadratic b-spline surface.*
- *Extraordinary points: possibly "ugly" behaviour, e.g. no proper tangent plain.*

Catmull-Clark Subdivision Surface

Property (Catmull-Clark Subdivision Surface)

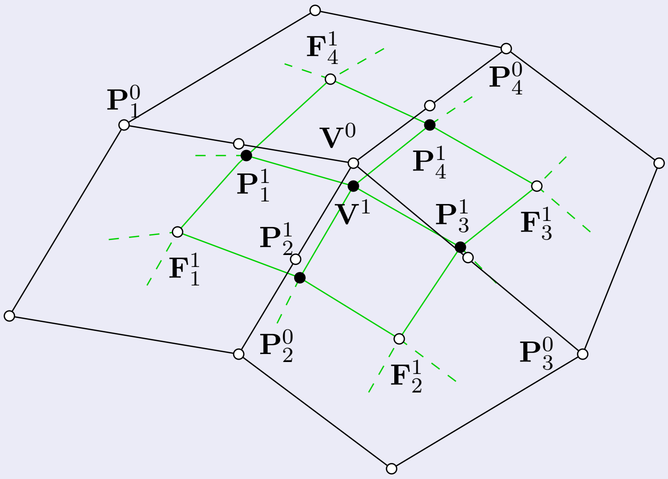
Given: *2-mannifold Polygon Mesh* $\mathcal{P}^0$

Subdivision Rule: *Consider vertex* $\mathbf{V}^0$ *(valence* k*), neighboring vertices* $\mathbf{P}_i^0$ *and center points* $\mathbf{F}_i^1,\ i = 1, \ldots, k$ *of the neighboring polygons.*

edge point: $\mathbf{P}_i^1 = \frac{\mathbf{V}^0 + \mathbf{P}_i^0 + \mathbf{F}_i^1 + \mathbf{F}_{i-1}^1}{4}$

New position of the vertex:

$$\mathbf{V}^1 = \frac{k-2}{k}\mathbf{V}^0 + \frac{1}{k^2}\sum_{i=1}^{k}\mathbf{P}_i^0 + \frac{1}{k^2}\sum_{i=1}^{k}\mathbf{F}_i^1$$



In each step:

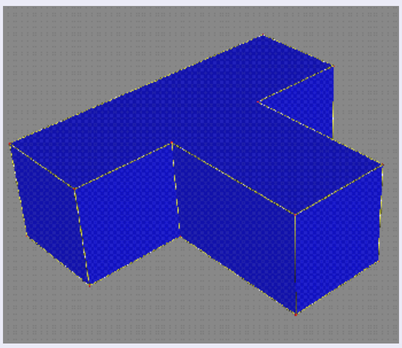
k*-sided polygon* $\Rightarrow$ k *quadrangle*

$\left.\begin{array}{l}k\text{-valent vertex}\\ k\text{-sided polyg.}\end{array}\right\} \Rightarrow k\text{-valent vert.}$

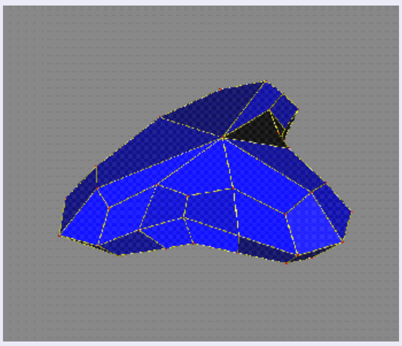
Properties of the Catmull-Clark Subdivision Surface

Property (Catmull-Clark Subdivision Scheme)

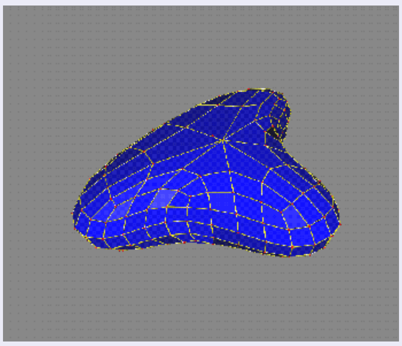
1. *each* k*-sided polygon results in* k *quadrangles*
2. *Catmull-Clark surfaces are a generalization of cubic b-spline surfaces*
3. k*-valent vertice/*k*-sided surface,* $k \neq 4$ *deliver for* $\mathcal{P} = \mathcal{P}^\infty$ *extraordinary points:*
   - *Outside of these points is:* $\mathcal{P}$ *a local* C^2*-steady, cubical b-spline surfaces.*
   - *Extraordinary points: possibly "ugly" behaviour (low consistency)*



Initial mesh



one subdivision



two subdivisions

Approach (Loop Subdivision Surface (Ch. Loop 87))

Given: *2-mannifold triangle mesh* $\mathcal{P}^0$

Subdivision Rule: *Consider vertex* $\mathbf{V}^0$ *(valence* k*) and neighboring vertices* $\mathbf{P}_i^0,\ i=1,\ldots,k$

New vertex position:

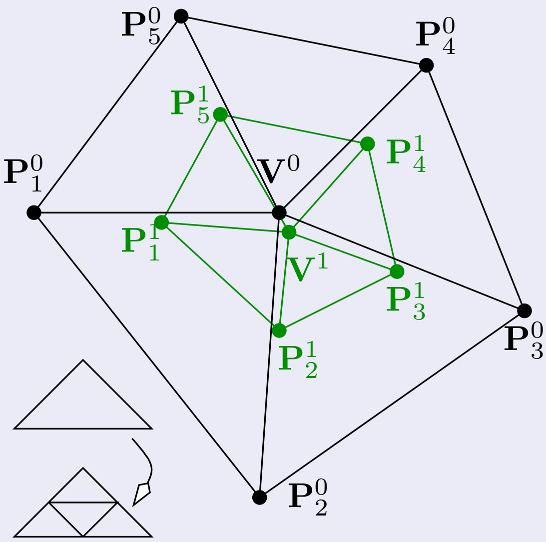
$$\mathbf{V}^1 = (1-\beta)\mathbf{V}^0 + \beta \cdot \frac{1}{k}(\mathbf{P}_1^0 + \ldots \mathbf{P}_k^0)$$

$$\beta = \frac{5}{8} - \frac{(3+2\cos(2\pi/k))^2}{64}$$

Edge center:

$$\mathbf{P}_i^1 = \frac{3\mathbf{V}^0 + 3\mathbf{P}_i^0 + \mathbf{P}_{i-1}^0 + \mathbf{P}_{i+1}^0}{8}$$

In each step:
Four new triangles for each triangle



Property (Loop-Subdivision Scheme)

1. *each triangle is subdivided into four triangles*
2. *Extraordinary points result from vertices with valence* $\neq 6$ *(regular valence* 6*):*
   - *Outside of these points is:* $\mathcal{P}$ *local* C^2*-steady*
   - *Extraordinary points: Possibly "ugly" behaviour (reduced continuity)*