Assignment in Computer Graphics II

– Assignment 9 –

Computer Graphics and Multimedia Systems Group

David Bulczak, Christoph Schikora

Assignment 1 [2 Points] Solid Modeling

To achieve boolean operations with polygonal b-reps, efficient computations for polygon intersections are necessary.

Given two planar polygons with vertices $\{\mathbf{P}_1, \ldots, \mathbf{P}_k\}$ respectively $\{\mathbf{Q}_1, \ldots, \mathbf{Q}_l\}$ and two planes E_P, E_Q containing the polygons.

E_P, E_Q are given in Hesse normal form:

$$E_P: \quad \hat{\mathbf{n}}_P \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} + d_P = 0, \quad \hat{\mathbf{n}}_P = \begin{pmatrix} a_P \\ b_P \\ c_P \end{pmatrix}$$

$$E_Q: \quad \hat{\mathbf{n}}_Q \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} + d_Q = 0, \quad \hat{\mathbf{n}}_Q = \begin{pmatrix} a_Q \\ b_Q \\ c_Q \end{pmatrix}$$

1. Show that for arbitrary vectors

$$\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \ \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}, \ \mathbf{c} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

the following holds:

$$\mathbf{b} \times \mathbf{a} = -\mathbf{a} \times \mathbf{b}, \tag{1}$$

$$\mathbf{a} \times \mathbf{a} = 0, \tag{2}$$

$$\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b}) = 0, \tag{3}$$

$$\mathbf{a} \times (\beta\mathbf{b} + \gamma\mathbf{c}) = \beta(\mathbf{a} \times \mathbf{b}) + \gamma(\mathbf{a} \times \mathbf{c}) \tag{4}$$

with scalar factors $\beta, \gamma \in \mathbb{R}$.

2. To implement the intersection computation we need a function `intersectPlane`, that intersects two planes E_P, E_Q and returns a line G in parametric form.
   Proof that

$$G : \mathbf{P} + \alpha\vec{\mathbf{l}}, \alpha \in \mathbb{R} \text{ with } \mathbf{P} = \frac{(d_Q\hat{\mathbf{n}}_P - d_P\hat{\mathbf{n}}_Q) \times (\hat{\mathbf{n}}_P \times \hat{\mathbf{n}}_Q)}{\|\hat{\mathbf{n}}_P \times \hat{\mathbf{n}}_Q\|^2} \text{ and } \vec{\mathbf{l}} = \hat{\mathbf{n}}_P \times \hat{\mathbf{n}}_Q$$

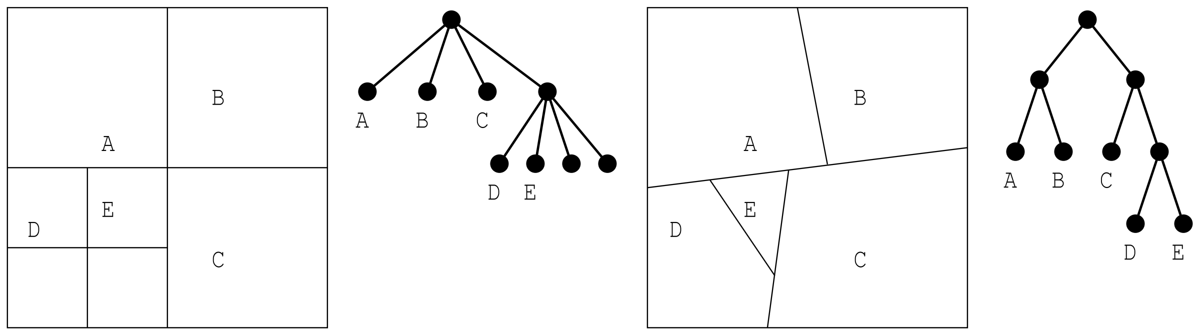
   solves the problem!

   Hint:

   1. **Think about the properties of G w.r.t. the planes.**
   2. $\hat{\mathbf{a}} \cdot (\hat{\mathbf{b}} \times (\hat{\mathbf{c}} \times \hat{\mathbf{d}})) = (\hat{\mathbf{a}} \times \hat{\mathbf{b}}) \cdot (\hat{\mathbf{c}} \times \hat{\mathbf{d}})$

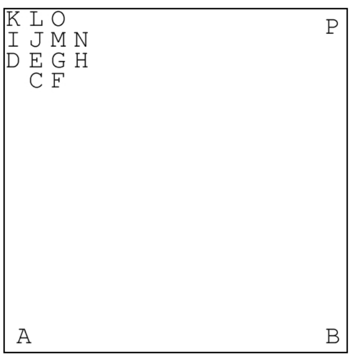
Assignment 1 [2 Points] Quadtree vs. BSP Tree

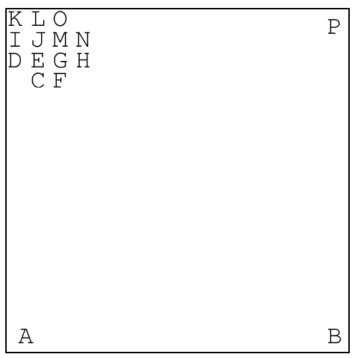
Create schematically both, a quadtree and a BSP tree on the basis of a given space with positioned objects ($\mathbf{A}$ - $\mathbf{P}$). When creating the BSP-Trees, the subdivisions should be made in the way that the divided space divides objects into two subspaces of equal numbers. The subdivision has to be repeated until only one object per segment is present. Example:



[1.]

Draw the quadtree and the BSP partitioning, and specify the corresponding trees for the following objects:





[2.]

Evaluate your results: What can be said about the complexity of locating an object within a Tree? Note: Consider possible extreme situations for the distribution of objects in the room!

Assignment 3 [2 Points] Complex numbers

Given two complex numbers $p = 2+3i$ und $q = 4-i$ determine $p+q$ and $p \cdot q$.

Let be $c = a+ib \in \mathbb{C}$ a complex number.
(Reminder: $Im(c) = b$ is the imaginary part and $Re(c) = a$ is the real part of c). Proof the following equations:

$$Re(c) = \frac{1}{2}(c+\overline{c})$$

$$Im(c) = \frac{1}{2i}(c-\overline{c})$$

Note: For a given complex number $c = a+ib \in \mathbb{C}$ the conjugate complex number $\overline{c}$ is defined as
$\overline{c} = a-ib \in \mathbb{C}$.

Determine the real and imaginary part of the following term:

$$\frac{1}{1+i}$$

Simplify the term i^{33} as far as possible.

Assignment 4 [1 Point] Complex numbers (Bonus task)

Show that the product of two complex numbers $c_1,\ c_2 \in \mathbb{C}$ with $|c_1| = |c_2| = 1$ holds: $|c_1 \cdot c_2| = 1$.

Hand in: Until 28.06.2017 12:00 o'clock in mailbox of our chair (next to room H-A 7107).