Assignment in Computer Graphics II

– Assignment 10 –

Computer Graphics and
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Assignment 1 [2 Points] Quaternion

Given the following quaternions

$$\mathbf{q_1} = \tfrac{5}{13} - \tfrac{12}{13}k \qquad \mathbf{q_2} = \tfrac{4}{5} + \tfrac{4}{5}j \qquad \mathbf{q_3} = \tfrac{1}{17} - \tfrac{12}{17}i + \tfrac{12}{17}j$$

and let be $s_{ij} := q_i + q_j$ and $p_{ij} := q_i q_j$ the sum and the product of quaternions.

1. **Addition:** Calculate s_{12}, s_{23} and s_{13}.
2. **Multiplication:** Calculate p_{12} and p_{13}.
3. Determine if $q_1, q_2, q_3 s_{12}, s_{23}, s_{13}, p_{12}$ and p_{13} correspond to rotations in 3D.

Assignment 2 [2 Points] Rotation

Name the quaternion that corresponds to an rotation by angle π around the z-axis.

1. Convert the quaternion into a rotation matrix.
2. Show that both representations are equal for $\mathbf{w} = \begin{pmatrix} 0 & 4 & 2 \end{pmatrix}^T$

Annotation: Please indicate in each case the complete solution.

Assignment 3 [2 Points] Arc length

Given the curve $\mathbf{C}(u) = \begin{pmatrix} u \\ \sqrt{1-u^2} \end{pmatrix}$ in R^2 for $u \in [0,1]$. [1.]

Sketch the shape of the curve.

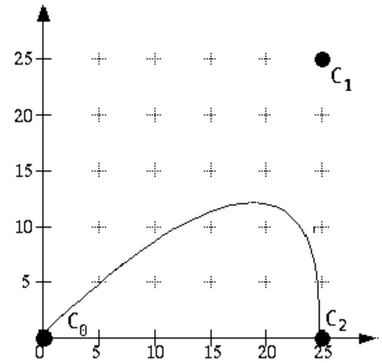
Calculate the arc length function $l_C(u)$ with the help of $\mathbf{C}'(u)$. Use the equation

$$\frac{d}{dx} arcsin(x) = \frac{1}{\sqrt{1-x^2}}. \tag{1}$$

Curve $\mathbf{C}$ should be traversed with constant velocity in the time interval $t \in [0,1]$. Give the function $s(t)$, which describes the traveled distance for time interval $t \in [0,1]$. Calculate $\mathbf{C}(t)$ with the function $s(t)$.

Assignment 4 [2 Points] Spline-based animation

Given the Bezier curve $\mathbf{C}(u)$ (shown in the figure) with control points
$\mathbf{C}_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\mathbf{C}_1 = \begin{pmatrix} 25 \\ 25 \end{pmatrix}$ und $\mathbf{C}_2 = \begin{pmatrix} 25 \\ 0 \end{pmatrix}$.



Also given curve points for $u_1 = 0.2,\ u_2 = 0.4,\ u_3 = 0.6,\ u_4 = 0.8$:

$$\mathbf{C}(u_1) = \begin{pmatrix} 9 \\ 8 \end{pmatrix},\ \mathbf{C}(u_2) = \begin{pmatrix} 16 \\ 12 \end{pmatrix},\ \mathbf{C}(u_3) = \begin{pmatrix} 21 \\ 12 \end{pmatrix},\ \mathbf{C}(u_4) = \begin{pmatrix} 24 \\ 8 \end{pmatrix}.$$

Two lookup tables with arc lengths have to be completed:

u_i	*Bogen*
$u_0 = 0$	$l_0 = 0$
$u_1 = 0{,}2$	$l_1 =$
$u_2 = 0{,}4$	$l_2 =$
$u_3 = 0{,}6$	$l_3 =$
$u_4 = 0{,}8$	$l_4 =$
$u_5 = 1{,}0$	$l_5 =$

Bogen	u_i^*
$l_0^* = 0$	$u_0^* = 0$
$l_1^* =$	$u_1^* =$
$l_2^* =$	$u_2^* =$
$l_3^* =$	$u_3^* =$
$l_4^* =$	$u_4^* =$
$l_5^* =$	$u_5^* =$

[1.]

For Parameters $u_0, \ldots, u_5$ compute respectively the approximations l_i between $\mathbf{C}(u_0)$ und $\mathbf{C}(u_i)$. Insert the values into the table.

Divide the total length into five equidistant parts and insert the interim values $l_1^*, \ldots, l_5^*$ into the table.

Determine for the arc lengths $l_1^*, \ldots, l_5^*$ the corresponding parameters $u_1^*, \ldots, u_5^*$, to get the curve points in equidistant distances.
Hint: For every arc length perform a search in the left table. If the found value is between two table entries then use linear interpolation to obtain the new parameter.

Hand in: Until 06.07.2017 12:00 o'clock in mailbox of our chair (next to room H-A 7107).