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Assignment in Computer Graphics II

- Assignment 9 -Computer Graphics and Multimedia Systems Group Markus Kluge, Dmitri Presnov

Assignment 1 [1 Point] Body of revolution

Consider the function $y = f(x) = \sqrt{1 - x^2}$ with $x \in [-1, 1]$.

Calculate the corresponding body of revolution obtained by the rotation about the x-axis function. Calculate surface area and volume of the rotating body. Which geometric primitive corresponds to the body?

Annotation: Please indicate in each case the complete solution.

Assignment 2 [2 Points] Solid Modeling

To achieve boolean operations with polygonal b-reps, efficient computations for polygon intersections are necessary.

Given two planar polygons with vertices $\{\mathbf{P}_1, \dots, \mathbf{P}_k\}$ respectively $\{\mathbf{Q}_1, \dots, \mathbf{Q}_l\}$ and two planes E_P, E_Q containing the polygons.

 E_P, E_Q are given in Hesse normal form:

$$E_P: \quad \hat{\mathbf{n}}_P \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} + d_P = 0, \quad \hat{\mathbf{n}}_P = \begin{pmatrix} a_P \\ b_P \\ c_P \end{pmatrix}$$
$$E_Q: \quad \hat{\mathbf{n}}_Q \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} + d_Q = 0, \quad \hat{\mathbf{n}}_Q = \begin{pmatrix} a_Q \\ b_Q \\ c_Q \end{pmatrix}$$

1. Show that for arbitrary vectors

$$\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \ \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}, \ \mathbf{c} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

the following holds:

$$\mathbf{b} \times \mathbf{a} = -\mathbf{a} \times \mathbf{b},\tag{1}$$

$$\mathbf{a} \times \mathbf{a} = \mathbf{0},\tag{2}$$

$$\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b}) = 0, \tag{3}$$

$$\mathbf{a} \times (\beta \mathbf{b} + \gamma \mathbf{c}) = \beta(\mathbf{a} \times \mathbf{b}) + \gamma(\mathbf{a} \times \mathbf{c})$$
(4)

with scalar factors $\beta, \gamma \in \mathbb{R}$.

2. To implement the intersection computation we need a function intersectPlane, that intersects two planes E_P, E_Q and returns a line G in parametric form. Proof that

$$G: \mathbf{P} + \alpha \vec{\mathbf{l}}, \alpha \in \mathbb{R} \text{ with } \mathbf{P} = \frac{(d_Q \hat{\mathbf{n}}_P - d_P \hat{\mathbf{n}}_Q) \times (\hat{\mathbf{n}}_P \times \hat{\mathbf{n}}_Q)}{\|\hat{\mathbf{n}}_P \times \hat{\mathbf{n}}_Q\|^2} \text{ and } \vec{\mathbf{l}} = \hat{\mathbf{n}}_P \times \hat{\mathbf{n}}_Q$$

solves the problem!

Hint:

- **1.** Think about the properties of *G* w.r.t. the planes.
- **2.** $\hat{\mathbf{a}} \cdot (\hat{\mathbf{b}} \times (\hat{\mathbf{c}} \times \hat{\mathbf{d}})) = (\hat{\mathbf{a}} \times \hat{\mathbf{b}}) \cdot (\hat{\mathbf{c}} \times \hat{\mathbf{d}})$

Total points after sheet 9: 42 of 70.

Hand in: Until 14.06.2018 12:00 o'clock in mailbox of our chair (next to room 7115).