

Assignment in Computer Graphics II

– Assignment 11 –

Computer Graphics and Multimedia Systems Group

Markus Kluge, Dmitri Presnov, Jan Mußmann

Assignment 1 [2 Points] Rotation

Name the quaternion that corresponds to a rotation by angle π around the z-axis.

1. Convert the quaternion into a rotation matrix.
2. Show that both representations are equal for $\mathbf{w} = (0 \ 4 \ 2)^T$

Annotation: Please indicate in each case the complete solution.

Assignment 2 [2 Points] Arc length

Given the curve $\mathbf{C}(u) = \begin{pmatrix} u \\ \sqrt{1-u^2} \end{pmatrix}$ in R^2 for $u \in [0, 1]$.

1. Sketch the shape of the curve.
2. Calculate the arc length function $l_C(u)$ with the help of $\mathbf{C}'(u)$. Use the equation

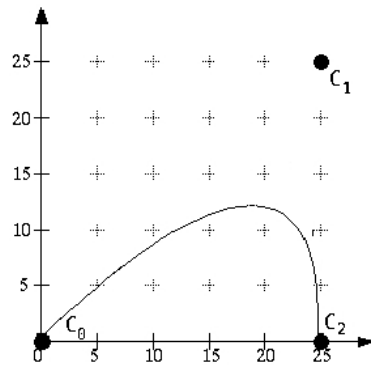
$$\frac{d}{dx} \arcsin(x) = \frac{1}{\sqrt{1-x^2}}. \quad (1)$$

3. Curve \mathbf{C} should be traversed with constant velocity in the time interval $t \in [0, 1]$. Give the function $s(t)$, which describes the traveled distance for time interval $t \in [0, 1]$. Calculate $\mathbf{C}(t)$ with the function $s(t)$.

Assignment 3 [2 Points] Spline-based animation

Given the Bezier curve $\mathbf{C}(u)$ (shown in the figure) with control points

$$\mathbf{C}_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \mathbf{C}_1 = \begin{pmatrix} 25 \\ 25 \end{pmatrix} \text{ und } \mathbf{C}_2 = \begin{pmatrix} 25 \\ 0 \end{pmatrix}.$$



Also given curve points for $u_1 = 0.2$, $u_2 = 0.4$, $u_3 = 0.6$, $u_4 = 0.8$:

$$\mathbf{C}(u_1) = \begin{pmatrix} 9 \\ 8 \end{pmatrix}, \mathbf{C}(u_2) = \begin{pmatrix} 16 \\ 12 \end{pmatrix}, \mathbf{C}(u_3) = \begin{pmatrix} 21 \\ 12 \end{pmatrix}, \mathbf{C}(u_4) = \begin{pmatrix} 24 \\ 8 \end{pmatrix}.$$

Two lookup tables with arc lengths have to be completed:

u_i	Bogen
$u_0 = 0$	$l_0 = 0$
$u_1 = 0,2$	$l_1 =$
$u_2 = 0,4$	$l_2 =$
$u_3 = 0,6$	$l_3 =$
$u_4 = 0,8$	$l_4 =$
$u_5 = 1,0$	$l_5 =$

Bogen	u_i^*
$l_0^* = 0$	$u_0^* = 0$
$l_1^* =$	$u_1^* =$
$l_2^* =$	$u_2^* =$
$l_3^* =$	$u_3^* =$
$l_4^* =$	$u_4^* =$
$l_5^* =$	$u_5^* =$

1. For Parameters u_0, \dots, u_5 compute respectively the approximations l_i between $\mathbf{C}(u_0)$ und $\mathbf{C}(u_i)$. Insert the values into the table.
2. Divide the total length into five equidistant parts and insert the interim values l_1^*, \dots, l_5^* into the table.
3. Determine for the arc lengths l_1^*, \dots, l_5^* the corresponding parameters u_1^*, \dots, u_5^* , to get the curve points in equidistant distances.

Hint: For every arc length perform a search in the left table. If the found value is between two table entries then use linear interpolation to obtain the new parameter.

Assignment 4 [2 Points] Form Control

Given the following Ease-function:

$$\text{ease}_{\text{exp}}(t) = \begin{cases} \frac{e^{2t} - 2t - 1}{2e - 4} & t \in [0, \frac{1}{2}] \\ 1 - \left(\frac{e^{2(1-t)} + 2t - 3}{2e - 4} \right) & t \in [\frac{1}{2}, 1] \end{cases}$$

1. Which velocity curve results from this Ease-function?
2. Let v_0 be a speed. Determine an Ease-function of the form

$$\text{ease}_{\text{exp}}^{v_0}(t) = c \cdot \text{ease}_{\text{exp}}(t),$$

which obtains an acceleration of 0 to v_0 in the time $t = 0$ to $t = \frac{1}{2}$, where c is a sought (gesuchter) constant term.

Note: It is sufficient to consider the definition on the interval $t \in [0, \frac{1}{2}]$.

Total points after sheet 11: 64 of 72.

Hand in: Until 27.06.2019 12:15 p.m. in mailbox of our chair (next to room 7115).