Medical Image Processing

Spatial Transformations Applied on Images

Important note:

- Please do not distribute any part of the materials or fully in any form or by any means.
- The materials privided here are just dedicated for MIP course in the University of Siegen and not for presentation.
- The imported medical images are from DICOM library:

http://www.osirix-viewer.com/resources/dicom-image-library/

Spatial Transformations Applied on Images

- Image interpolation methods

- Matrix transformation applying on the images

- Some appications

Image interpolation methods

- Mathematical Interpolation: estimating the value of a function at one or some positions between the function sample data (versus extrapolation)

- Important Techniques:
  1. Polynomial interpolation (linear and nonlinear)
  2. Exponential interpolaation
  3. Trigonometric interpolation
  4. Spline interpolation

Image interpolation methods

- The need to interpolation in image processing

1. Estimating the pixel values when resizing the image into a different size image or after applying some geometric transformations.

2. Estimating the pixel values when rotating the image with an arbitrary angle

3. Other applications (changing the image resolution).....

Image interpolation methods

- Main image interpolation categories

1. Adaptive image interpolation: the interpolation adapts the interpolated values in the image based on the other image information like edge, local gradient,... So, each pixel is treated individually (out of the scope of this course)

2. Non-adaptive image interpolation: all pixels are treated uniformly based on the underlying method (here we discuss about this categrory)

Image interpolation methods

- Non-adaptive image interpolation techniques:

1. Nearest neighbor interpolation: the value of the closest neighbor is taken for the new unknown pixel.
2. Linear interpolation: a first degree polynomial (basically a line) is used to connect between two known pixel points to estimate the point in between.
3. Bilinear interpolation: a 2-degree polynomial is used by getting the information from 4 closest neighbors.
4. Cubic interpolation: a third degree polynomial which uses the information of 16 pixel neighbors.

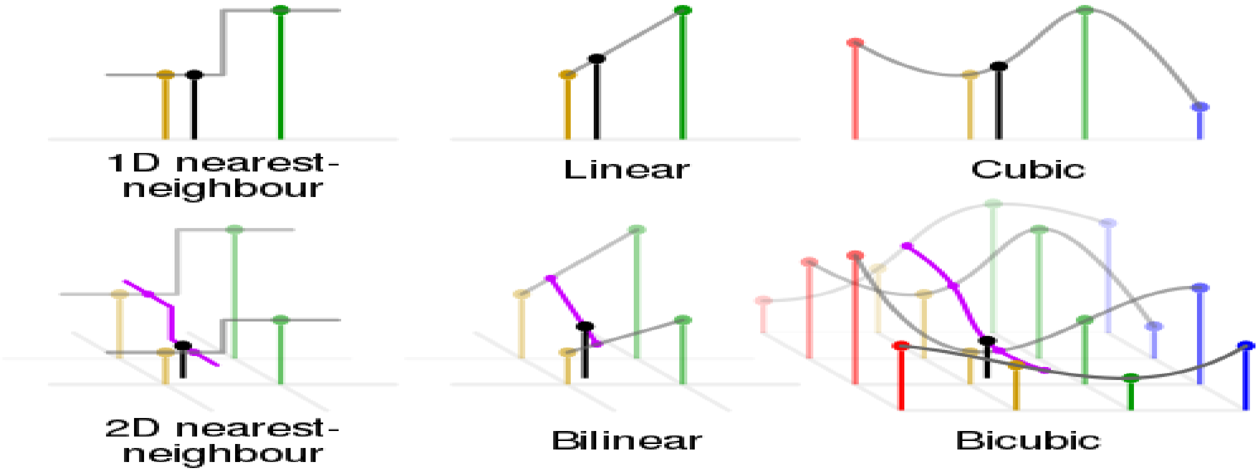
Image interpolation methods

- Other Non-adaptive image interpolation techniques:

  - B-spline interpolation
  - Sinc intepolation
  - Lanczos interpolation
  - ....

Image interpolation methods

- A quick look on the different interpolation methods:



(figure from: https://en.wikipedia.org/wiki/Nearest-neighbor_interpolation): black points are interpolating points and the other colors ara data sample

Image interpolation methods

- Applying resizing operation on the following image CT mage using different interpolation techniques:

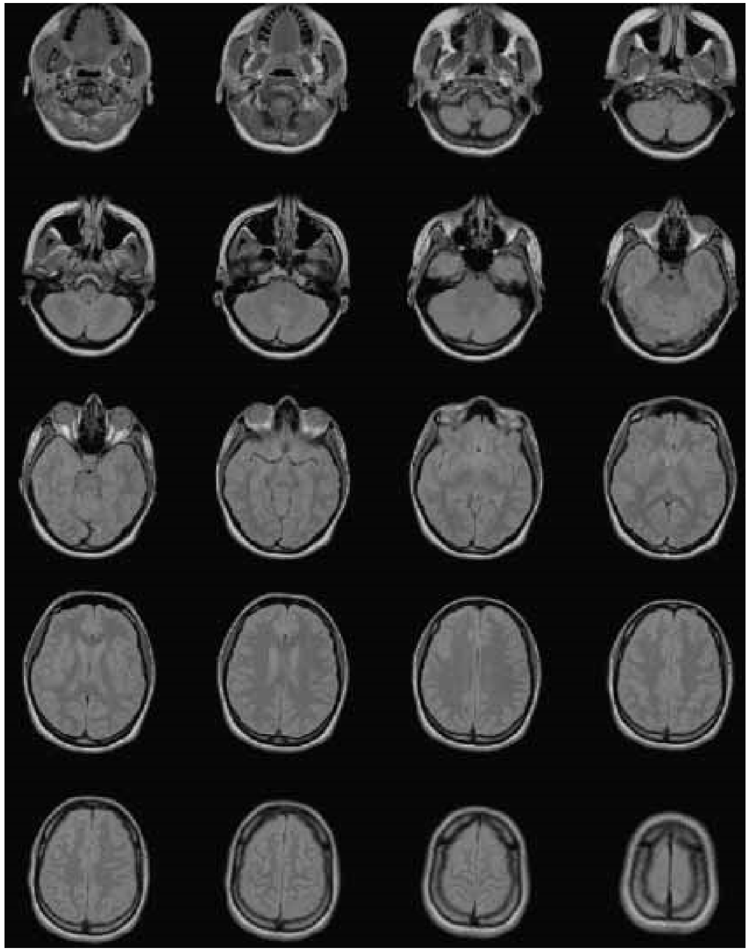


Image interpolation methods (resizing to 75%)

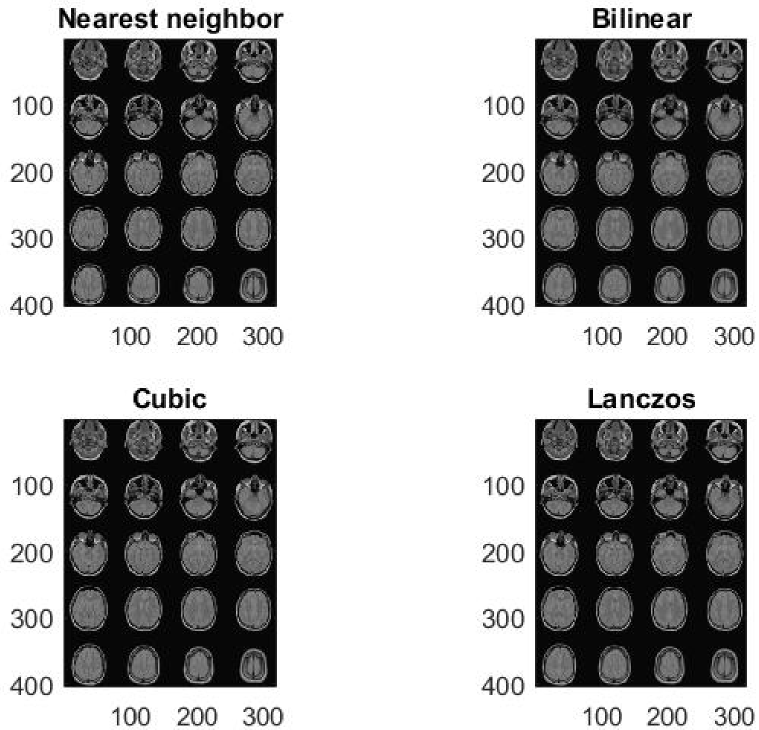
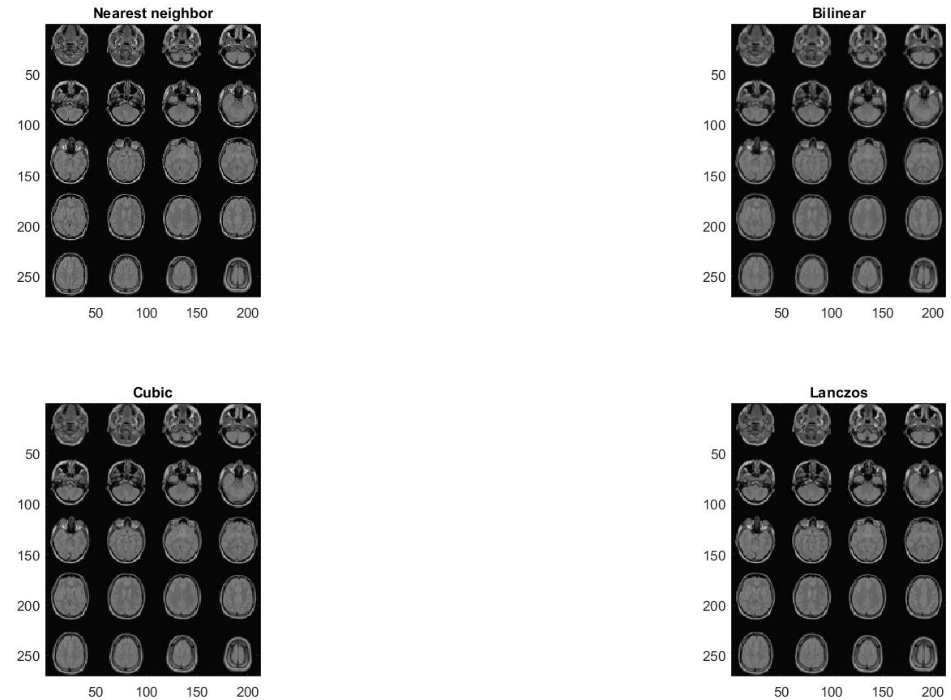


Image interpolation methods (resizing to 50%)



Matrix transformation applying on the images

- Image is represented as a matrix including all pixel intensities. So, the matrix transformations can be performed on the images.
- A transformation T maps pixel coordinates (x, y) to new coordinate (X, Y): T(x, y)=(X, Y)
  - An example:T(X, Y) = (cx, cy) resizes the image by a scaling factor of 1/c
- Basic geometric transformations can be presented as a matric indicating a 2-D linear transformation T:
  - Linearity: T(cV + W)=cT(V) + T(W) where, V and W are matrices and c an scalar
  - Example of some transformation: rotation, scaling, shearing, etc.

Matrix transformation applying on the images

- **Affine transformation:**
- One of the most important image transformation

- Affine transformation is a nlnlinear transformation

- Affine transformation is a mapping between two space which preserves points, straight lines and planes.

- An affine transformation is generally defined as a mapping from space X to space Y by f(x) = Mx+b where x is a vector in X and b a vector in Y. M also is a linear transformation in X which obviously can be presented as a matrix.

Matrix transformation applying on the images

- **Affine transformation (cont‘d):**
- As affine transformation y=Mx+b is not linear, it can not be presented by just by a matrix multiplication in general. So to resolve this issue an augmented matrix is built as follows:

$$\begin{bmatrix} y \\ 1 \end{bmatrix} = \begin{bmatrix} M & \vert b \\ 0\ 0 \ldots 0 & \vert 1 \end{bmatrix} \begin{bmatrix} x \\ 1 \end{bmatrix}$$

- In fact the equation introduced by the augmented matrix above is the same as original affine transformation y=Mx+b.

Matrix transformation applying on the images

- **Affine transformation (cont‘d):**
- Some important affine transformation with their matrix can be mentioned as below:

1. Translation transformation: $[x \quad y \quad 1] = [u \quad v \quad 1]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ T_u & T_v & 1 \end{bmatrix}$

2. Rotation transformatio: $[x \quad y \quad 1] = [u \quad v \quad 1]\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Matrix transformation applying on the images

- **Affine transformation (cont'd):**
- Some important affine transformation with their matrix can be mentioned as below:

3. Scaling transformation: $[x \quad y \quad 1] = [u \quad v \quad 1]\begin{bmatrix} S_u & 0 & 0 \\ 0 & S_v & 0 \\ 0 & 0 & 1 \end{bmatrix}$

4. Shearing (vertically) transformatio: $[x \quad y \quad 1] = [u \quad v \quad 1]\begin{bmatrix} 1 & 0 & 0 \\ S_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

5. Shearing (horizontally) transformatio: $[x \quad y \quad 1] = [u \quad v \quad 1]\begin{bmatrix} 1 & S_u & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Matrix transformation applying on the images

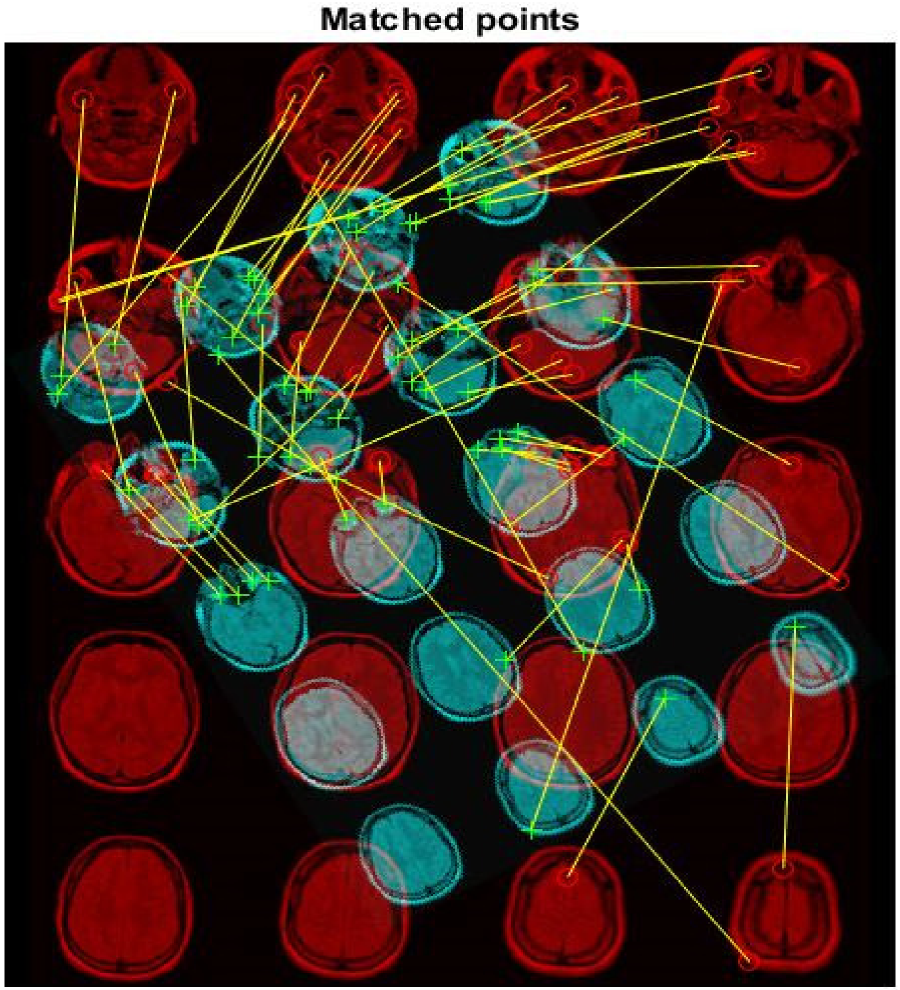
- **Composite transformations:**
- To apply composite transformations for example scaling+rotation, one must multiply the corresponding matrices to each other.

- After applying spatial transformation, the intensity values in the new pixel points are estimated by an interpolation method.

Some appications

- Image registration
- Image matching
- ....

Some appications: an example on image matching between an image and its rotated version (generated by MATLA image processing toolbox)



Useful links and references:

1. https://pdfs.semanticscholar.org/0050/6b20cede128dbec9c3bcb4d3f5a16a64bca9.pdf
2. http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.68.1891&rep=rep1&type=pdf
3. https://www.ldv.ei.tum.de/fileadmin/w00bfa/www/content_uploads/Vorlesung_3.2_SpatialTransformations.pdf