INSTITUTIONEN FÖR SYSTEMTEKNIK

LULEÅ TEKNISKA UNIVERSITET

Morphological Image Processing

Matthew Thurley

Industrial Image Processing

Last Lecture

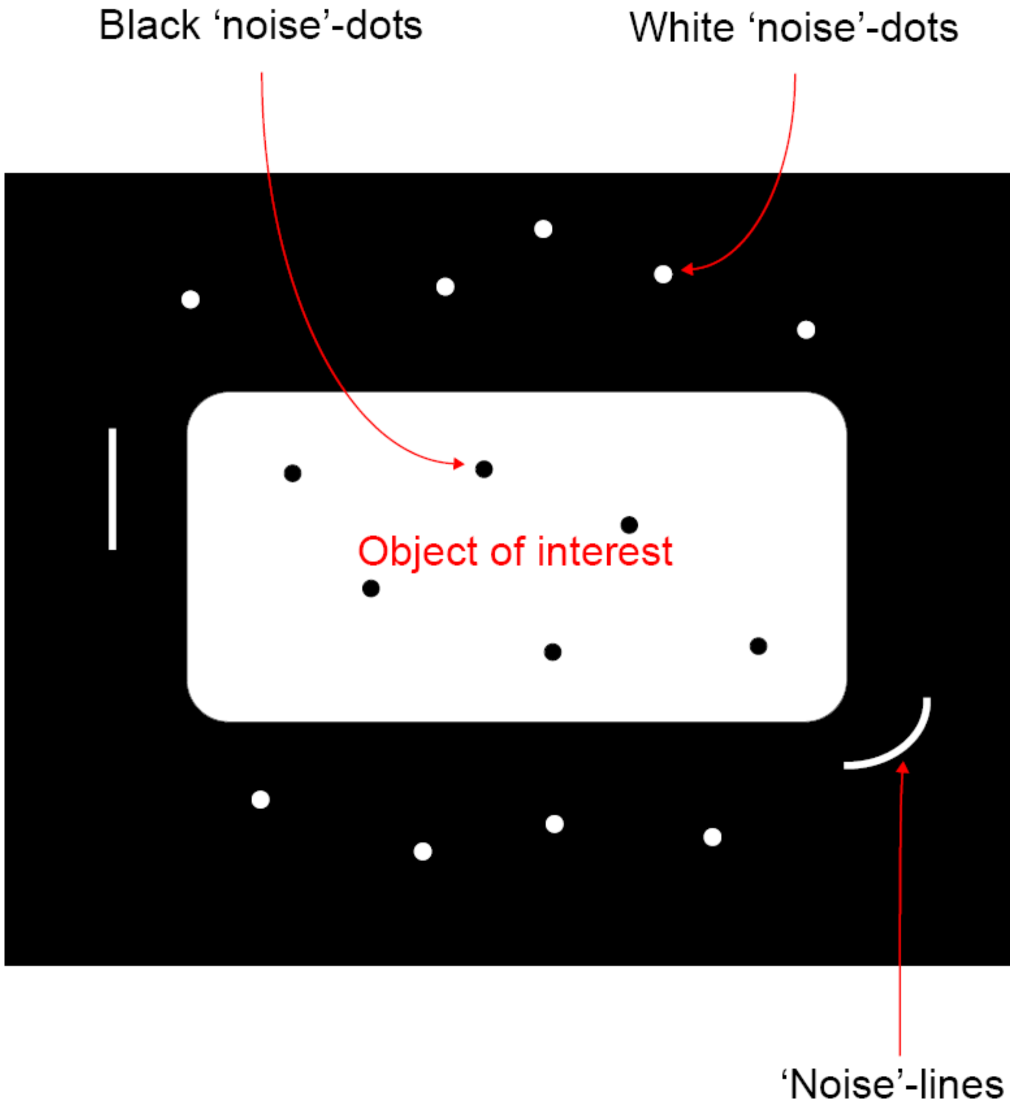
- Points, Lines, Edges
- More Neighbourhood Operations (Spatial Filters)
  - First Order Derivative Operators (Sobel)
  - Second Order (Laplacian)
  - Line Detection filters
  - First Order Compass Operators
  - Laplace of Gaussian Operator
- Image Sharpening
- Multiresolution Imaging: Pyramid Scheme

This Lecture

- Binary Morphological Image Processing
  - Erosion & Dilation
  - Opening & Closing
  - Morphological Boundary Detection
  - Connected Regions and Hole Filling
  - Watershed Segmentation
  - Distance Transform

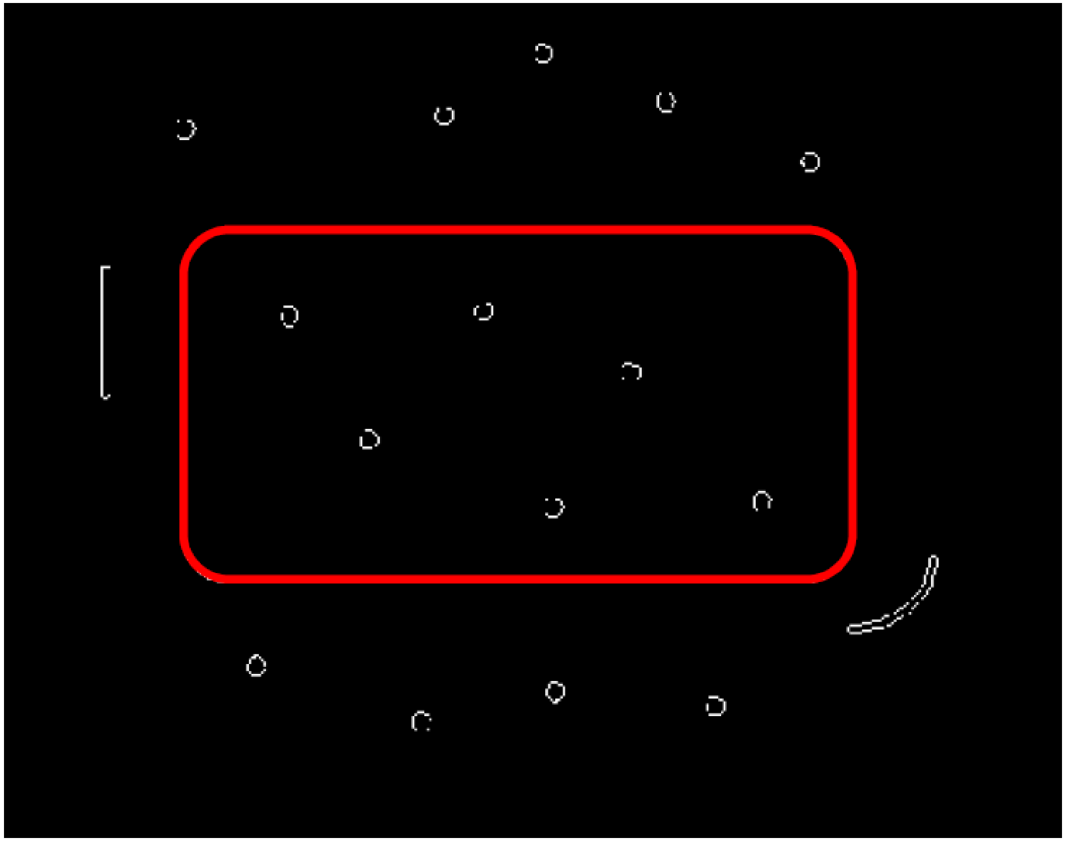
Binary Image Processing

- A thresholded image typically contains some objects of interest and some unwanted ’noise’.



Binary Image Processing

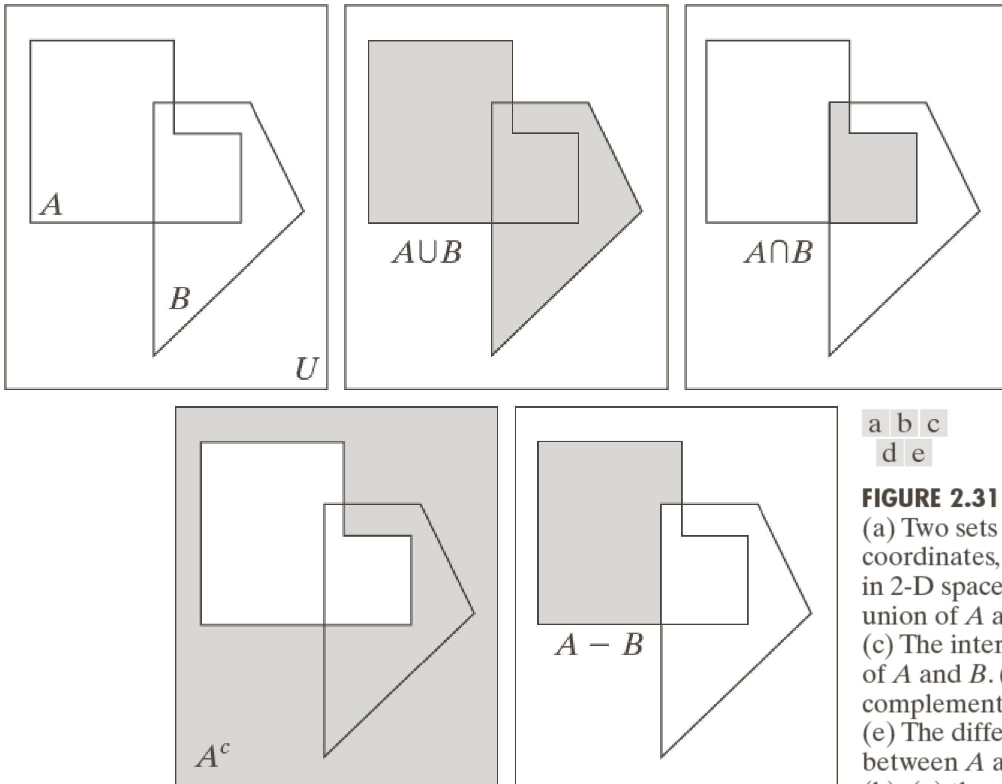
- A corresponding edge image will therefore contain ’edge noise’ (the edge of interest is shown in red)
- We introduce binary morphological image processing in the context of this example



Morphological Image Processing

- The term *morphology* originates from the study of the shapes of plants.
- Mathematical morphology is concerned with the identification of geometric structure.
- It is a branch of non-linear image processing using neighborhood operations.
- Images are analysed in terms of shape and size using a *structuring element* (the neighbourhood)
- This is analogous to a sliding window operation where the structuring element is analogous to the window mask. However, where linear sliding window operations involve multiplication and addition, mophological processing involves set operations.
- The mathematical foundation is called Minkowski algebra.

Set Theory



a b c
d e

FIGURE 2.31
(a) Two sets of coordinates, A and B, in 2-D space. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B. In (b)–(e) the shaded areas represent the member of the set operation indicated.

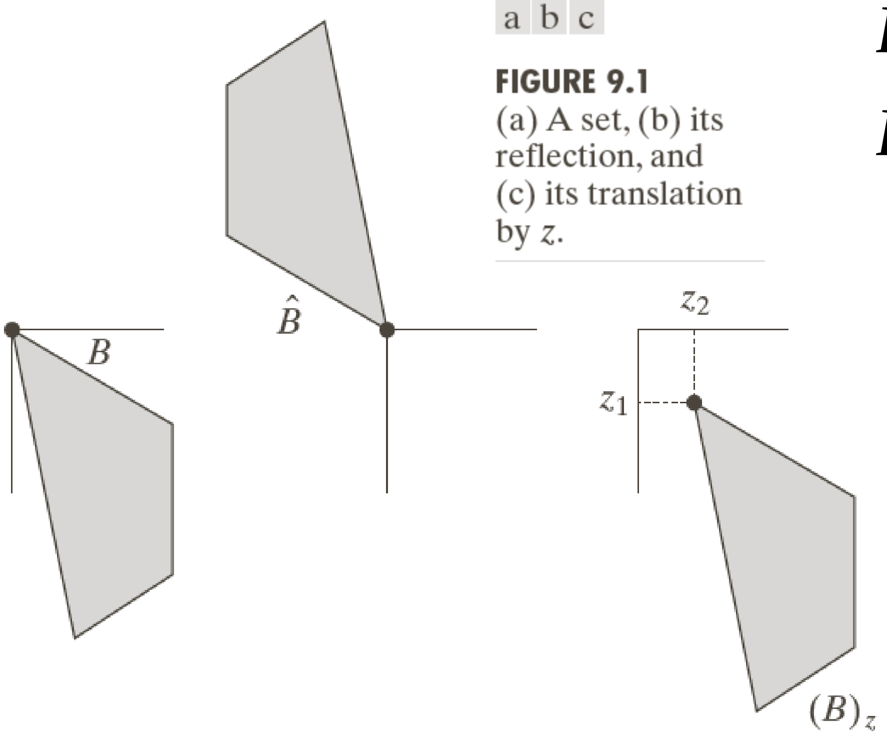
Set Theory	Binary Logic
$A \cup B$	A OR B
$A \cap B$	A AND B
A^c	NOT A
$A\text{-}B = A \cap B^c$	

$$\text{if } C = A \cap B$$
$$\Rightarrow C \subseteq B$$

Subset Operator

Set Theory

- For a given set we define the reflection and the translation.



a b c

FIGURE 9.1
(a) A set, (b) its reflection, and (c) its translation by z.

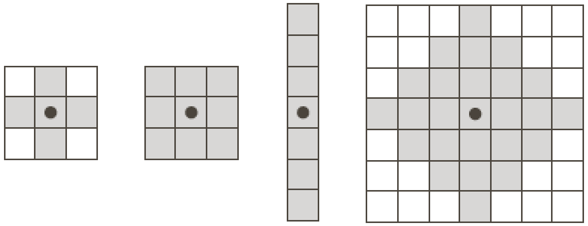
$$\hat{B} = \{-b : b \in B\}$$

$$B_z = \{b + z : b \in B\}, \mathrm{z} = (\mathrm{z}_1, z_2)$$

The reflection is the symmetric set with respect to the origin, all elements are scaled by -1.

Binary Morphological Image Processing

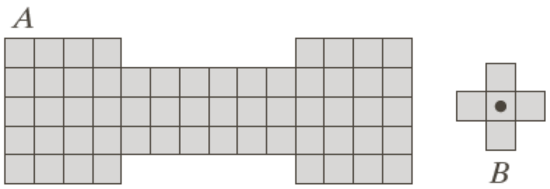
- A basic morphological operation can be described as *probing an image with a structuring elment (typically a small geometric shape) and quantifying how it fits or does not fit.*
- A structuring element is typically a small image or mask with an explicit origin (shown as the black dot).



- We now define the two fundamental morphological operations of *erosion* and *dilation.*

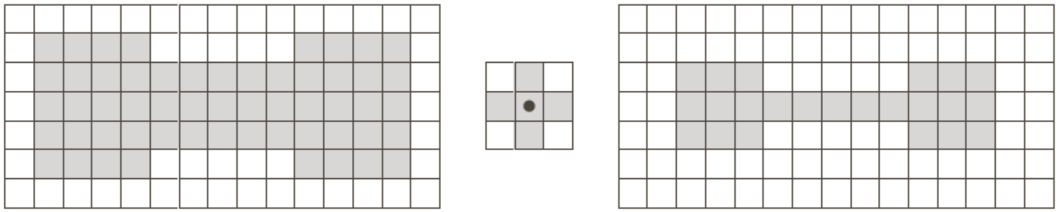
Binary Erosion

- Given an image A, and a structuring element B
- Recall B_z, the translation of B to some point z
- Define erosion as the set of points z, where B_z fits in the object A, that is B_z is a subset of A



$$B_z = \{b + z : b \in B\}$$

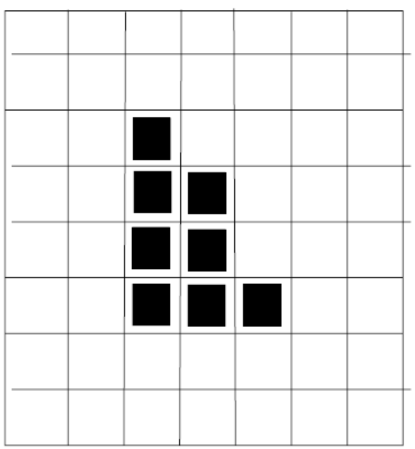
$$A \ominus B = \{z : B_z \subseteq A\}$$



- Erosion is not commutative $A \ominus B \neq B \ominus A$

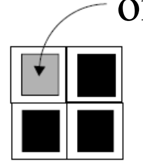
Binary Erosion

Binary Image A



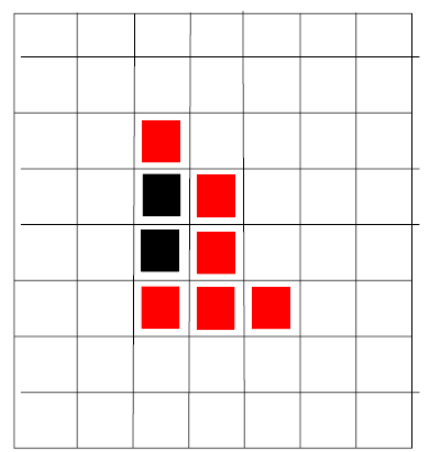
Structuring Element B

origin



$$B_z = \{b + z : b \in B\}$$

$$A \Theta B = \{z : B_z \subseteq A\}$$



Red points removed by the erosion

Binary Erosion

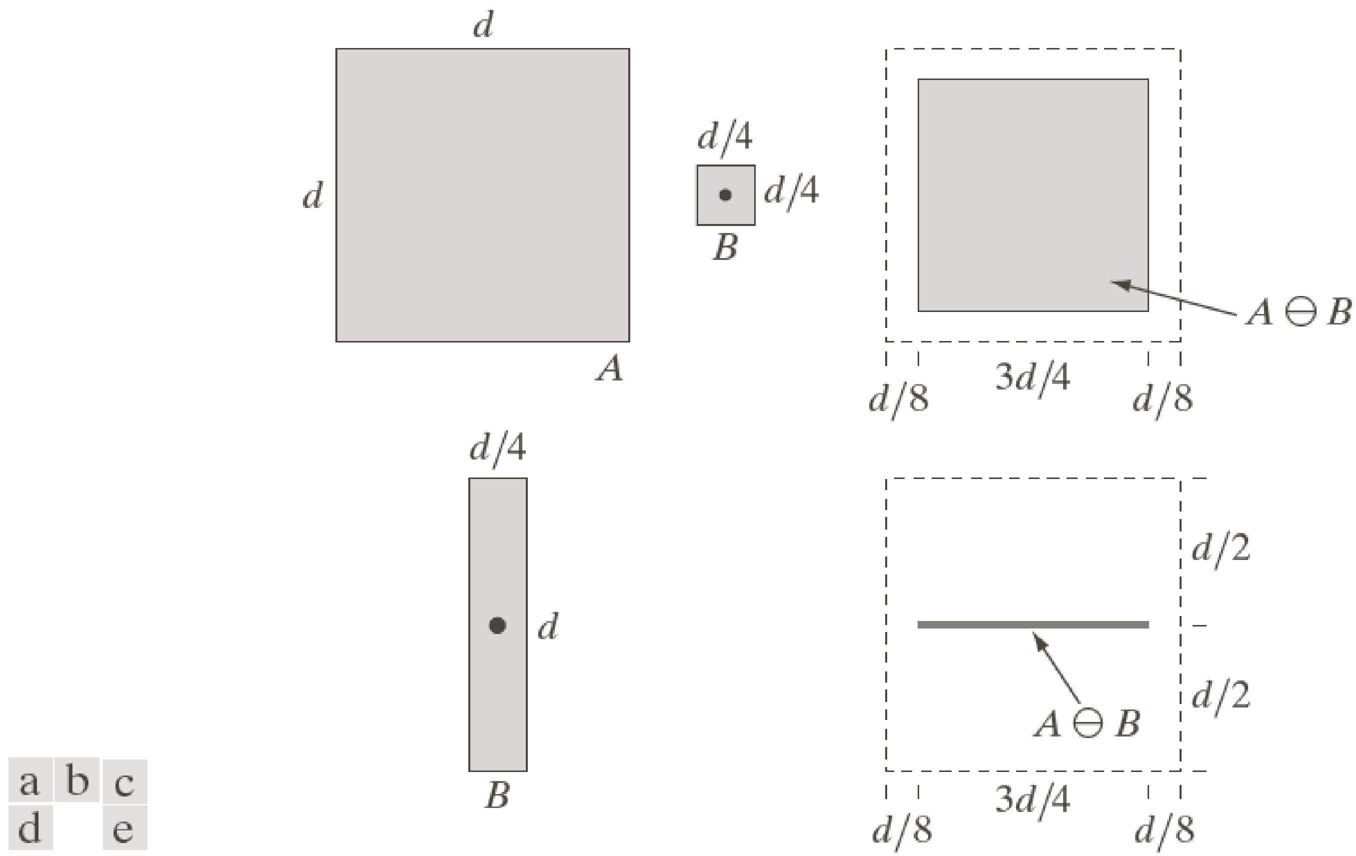


FIGURE 9.4 (a) Set A. (b) Square structuring element, B. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A by B using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference.

Binary Erosion

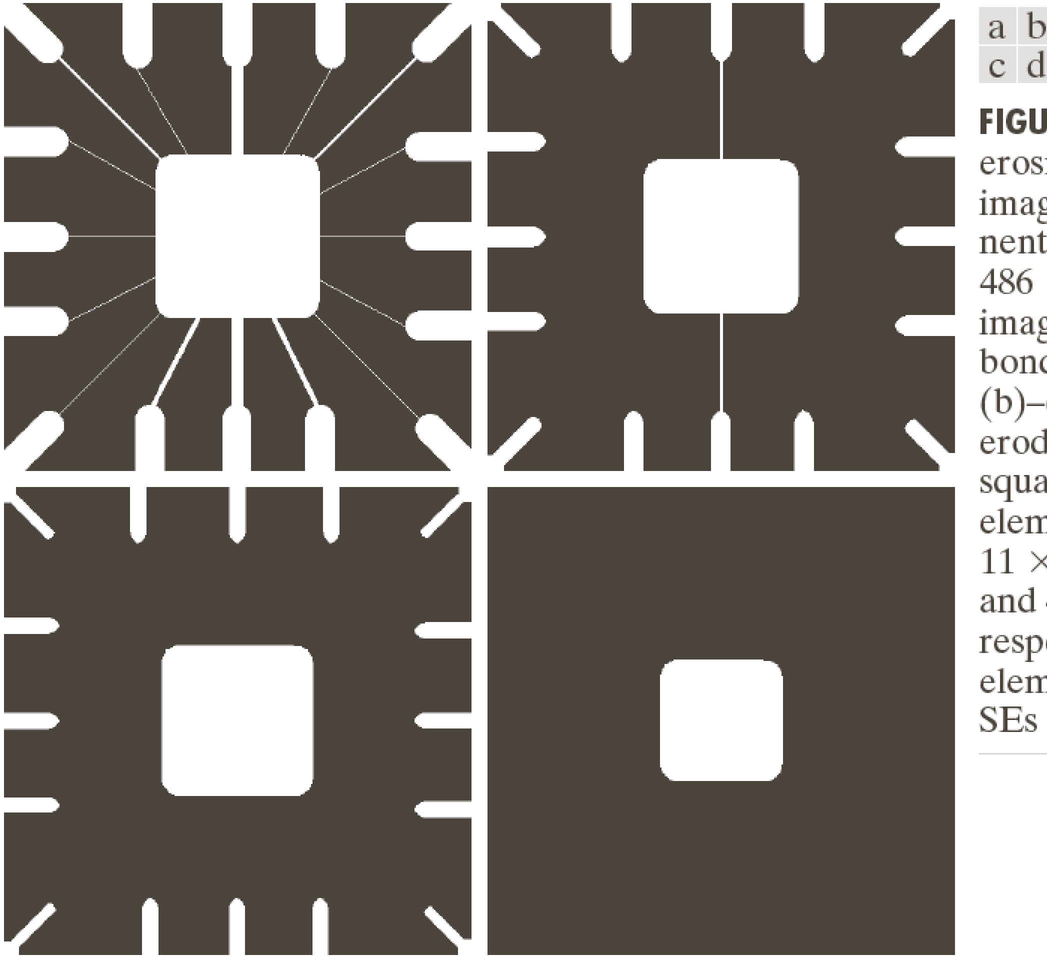
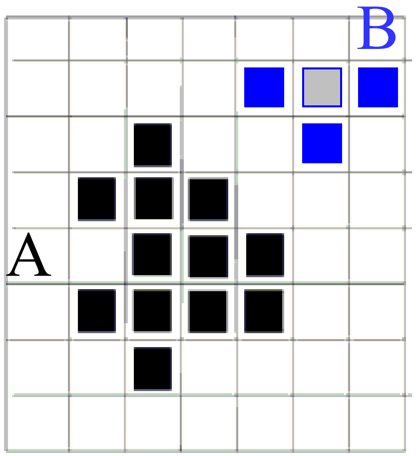


FIGURE 9.5 Using erosion to remove image components. (a) A 486×486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11×11, 15×15, and 45×45, respectively. The elements of the SEs were all 1s.

Quiz



- Peform the erosion of image A by structuring element B

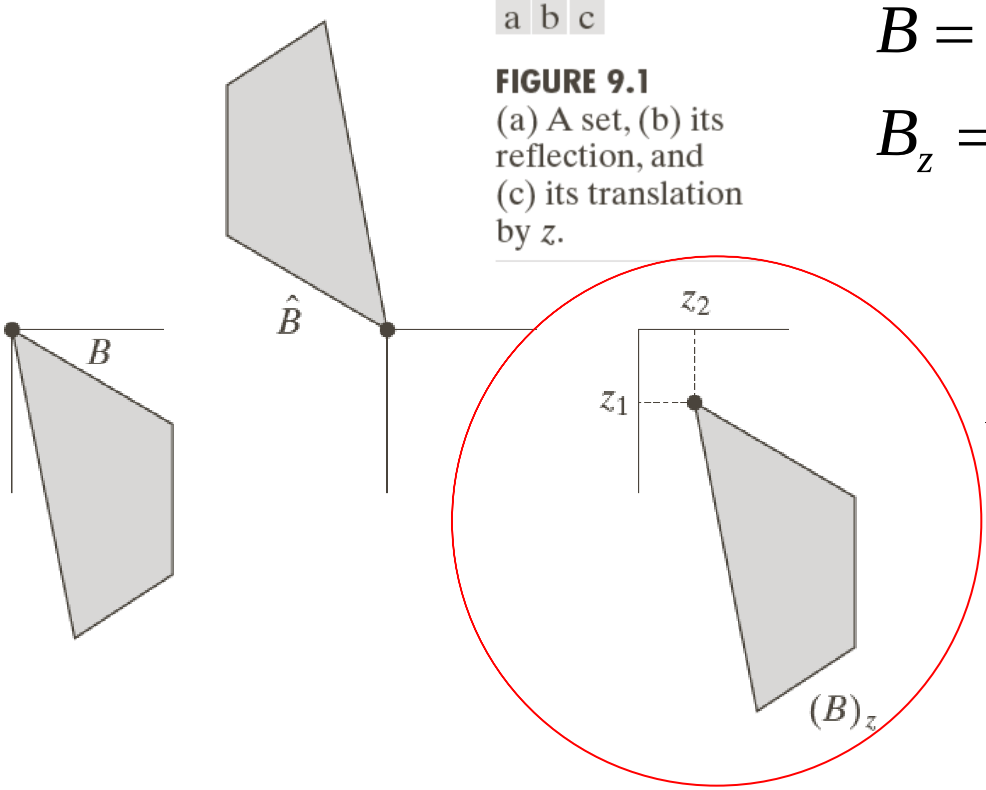
$$A \ominus B$$

- From question 9.6 in DIP textbook, perform the erosion of image A by B^2

$$A \ominus B^2$$

Set Theory

- For a given set we define the reflection and the translation.



a b c

FIGURE 9.1 (a) A set, (b) its reflection, and (c) its translation by z.

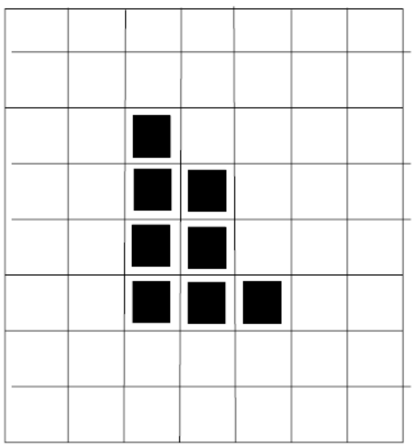
$$\hat{B} = \{-b : b \in B\}$$

$$B_z = \{b + z : b \in B\}, z = (z_1, z_2)$$

The reflection is the symmetric set with respect to the origin, all elements are scaled by -1.

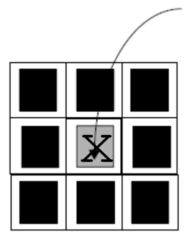
Binary Dilation

Binary Image A



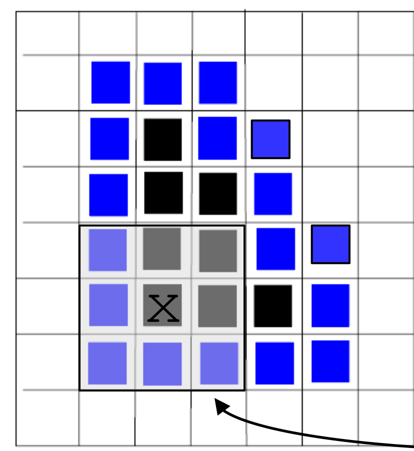
Structuring Element B

origin



$$B_z = \{b + z : b \in B\}$$

$$A \oplus B = \bigcup_{a \in A} B_a$$



Example structuring element position

Blue points added by the dilation

Imagine that the structuring element is a stamp, and you stamp the structuring element at each pixel in the image A

Dilation is commutative

$$A \oplus B = B \oplus A$$

Binary Dilation

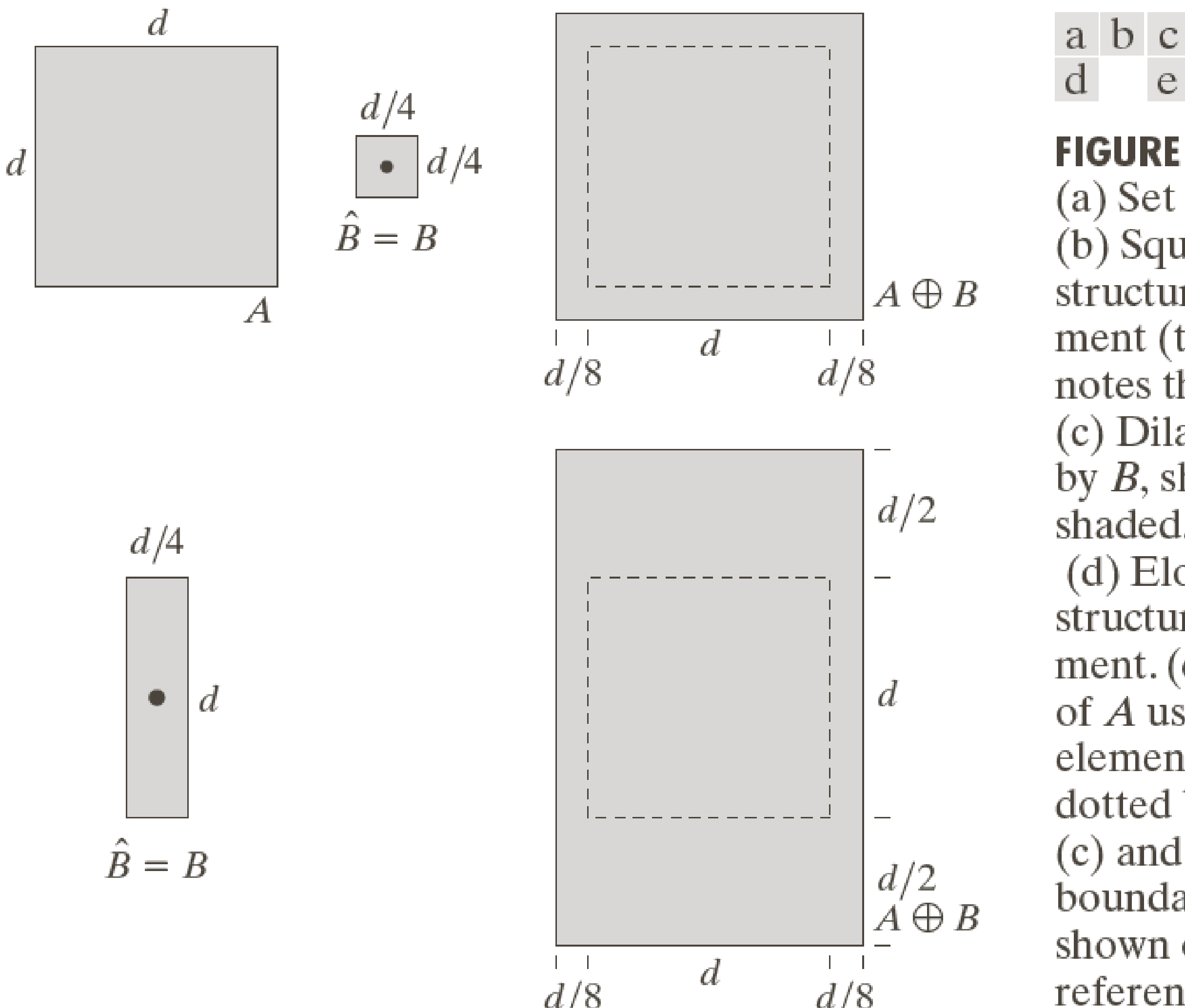
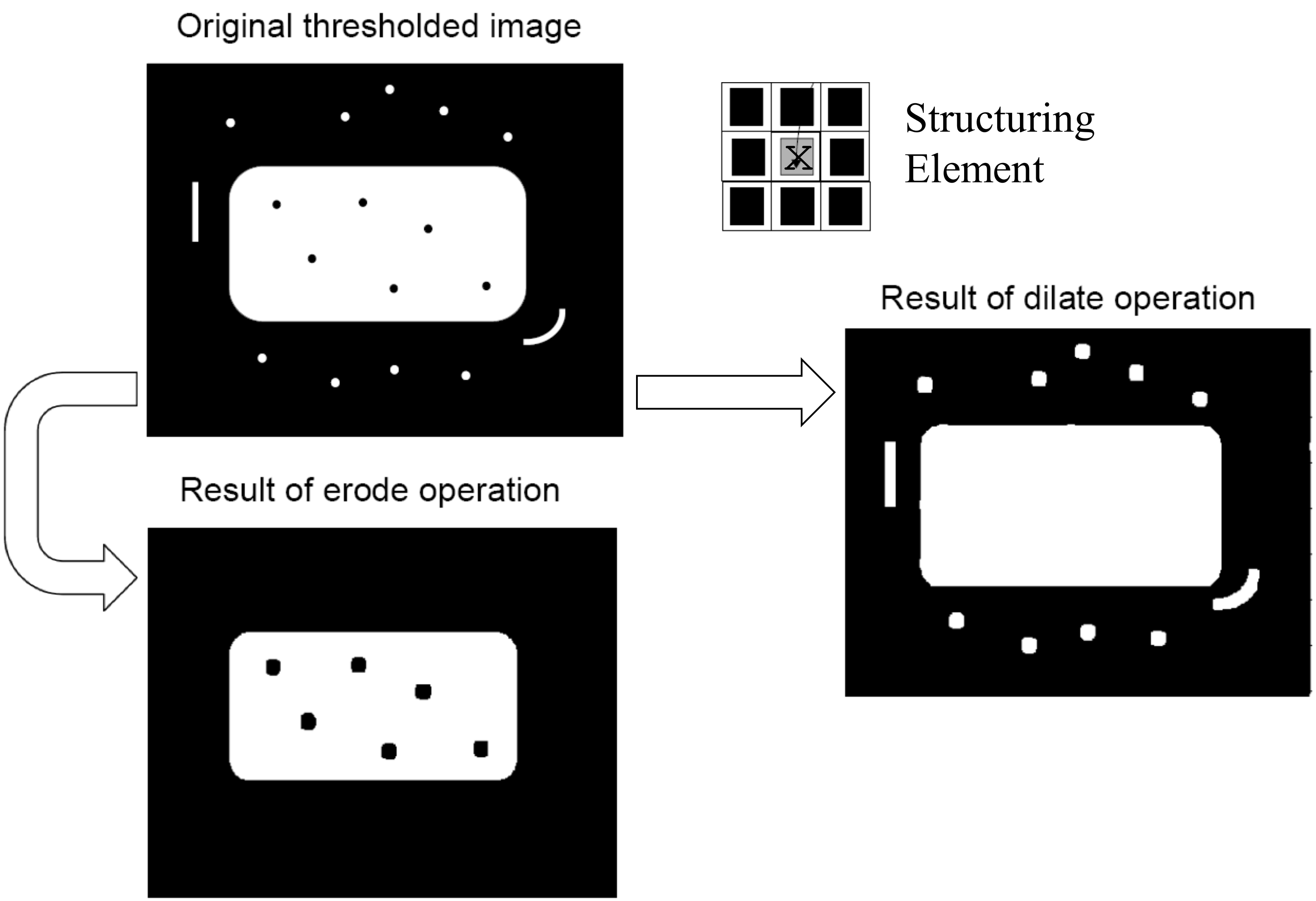
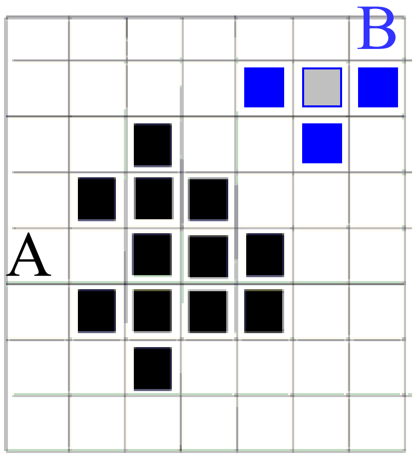


FIGURE 9.6
(a) Set A.
(b) Square structuring element (the dot denotes the origin).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element. (e) Dilation of A using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference

Binary Erosion and Dilation



Quiz



- Peform the following two dilations

$$A \oplus \hat{B}$$

$$A^c \oplus \hat{B}$$

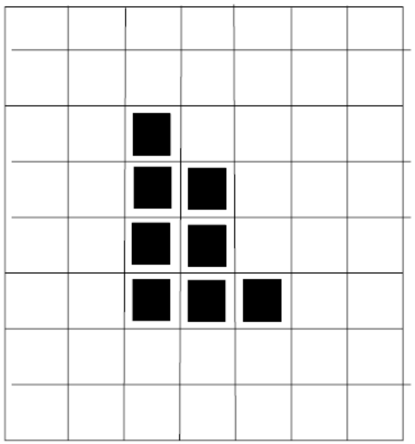
- Perform the dilation of image A by B^3 from question 9.6 in the DIP textbook

$$A \oplus B^3$$

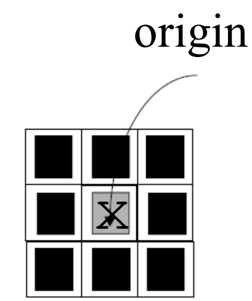
This definition of dilation is provided in the DIP textbook, section 9.2.2.

Alternative Definition - Binary Dilation

Binary Image A



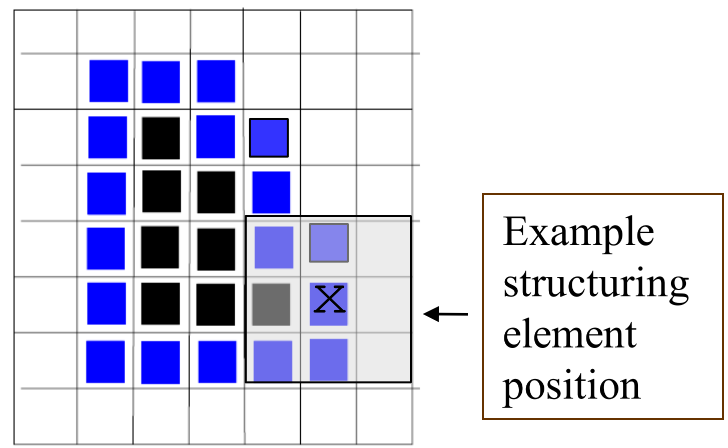
Structuring Element B



$$B_z = \{b + z : b \in B\}$$

$$\hat{B} = \{-b : b \in B\}$$

$$A \oplus B = \{z : (\hat{B})_z \cap A \neq null\}$$



Blue points added by the dilation

Binary Erosion and Dilation

- Dilation is commutative $X \oplus B = B \oplus X$
- Erosion is non-commutative $X \ominus B \neq B \ominus X$
- Dilation is associative $(X \oplus B) \oplus C = X \oplus (B \oplus C)$

Associativity

- A morphological operation is a neighbourhood operation
- The speed of morphological operations depends on the number of elements in the structuring element.
- A dilation by a 3 by 3 structuring element is faster than a dilation by a 5 by 5. The 3 by 3 has 9 elements and the 5 by 5 has 25 elements
- Let D be a 5 by 5 structuring element, B be a 3 by 3 structuring element, and C = B

$$\text{Observe that } D = B \oplus C$$

$$(X \oplus B) \oplus C \text{ requires fewer operations than } \mathrm{X} \oplus \mathrm{D}$$

- What if B is a 45 by 1 horizontal line, and C is a 1 by 45 vertical line, what is D?

Structuring Element Decomposition

- The process of decomposing a large structuring element (like D) into smaller structuring elements is known as structuring element decomposition. It can be used to perform operations with large structuring elements quickly by actually performing a series of smaller operations
- Revisit erosion. Erosion is not associative, but we can use structuring element decomposition just as effectively.

$$X \ominus D = X \ominus (B \oplus C) = (X \ominus B) \ominus C$$

- So instead of eroding by some big structureing element D, we could use a decomposition of D (D = B dilated by C), and instead erode by B and then erode by C
- This can be a lot faster. Consider if D is a 45 by 45 pixel square (2025 elements) and if B is a 45 by 1 vertical line, and C is a 1 by 45 horizontal line.

Binary Opening and Closing

- As we have seen, dilation expands the components of an image, while erosion shrinks them.
- We now introduce two additional operations derived from the fundamental operations of erosion and dilation.
- Opening generally smoothes the contour of objects, breaks narrow joins and eliminates thin protrusions.
- Closing also smoothes sections of the contour, but fuses narrow breaks, eliminates small holes and fills gaps in the contour.

Binary Opening

$$A \circ B = (A \ominus B) \oplus B$$

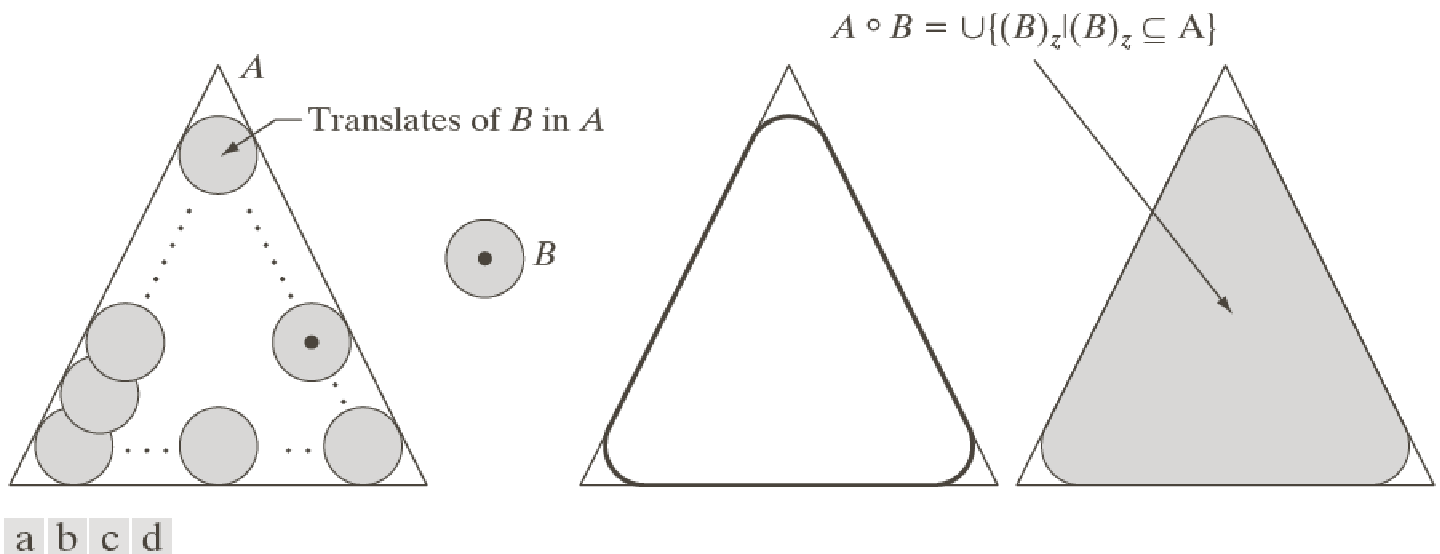
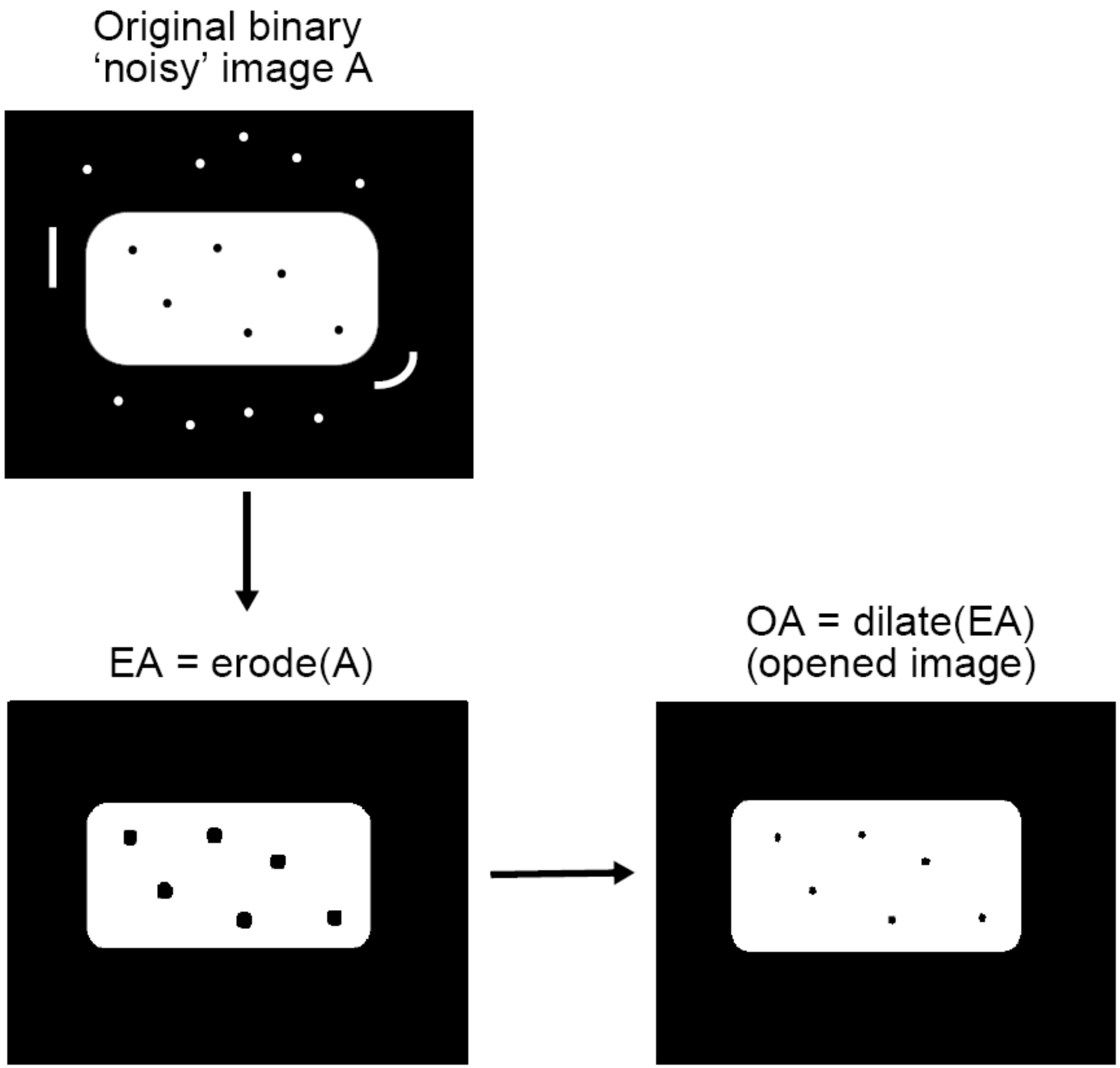


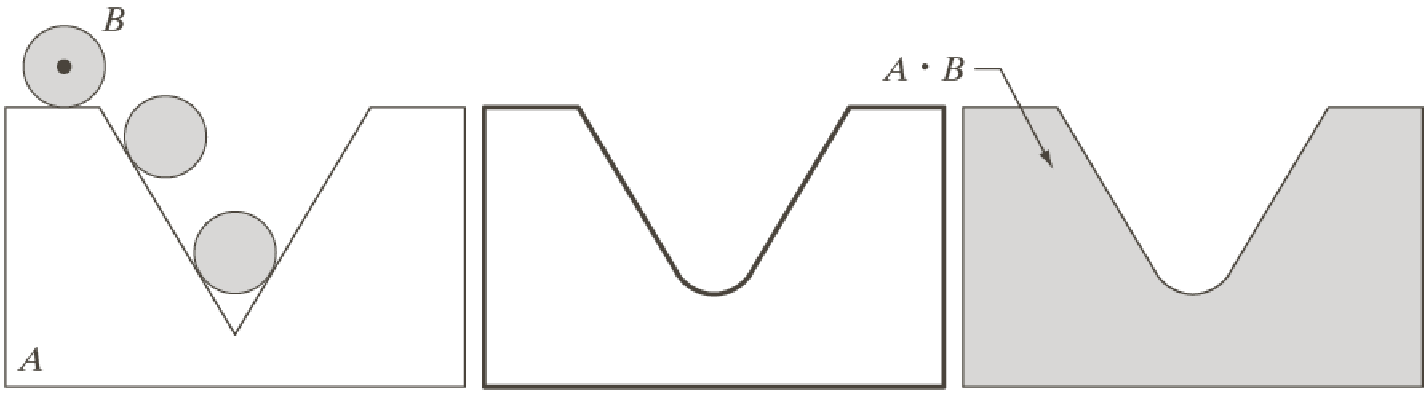
FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (b) Structuring element. (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded). We did not shade A in (a) for clarity.

Binary Opening



Binary Closing

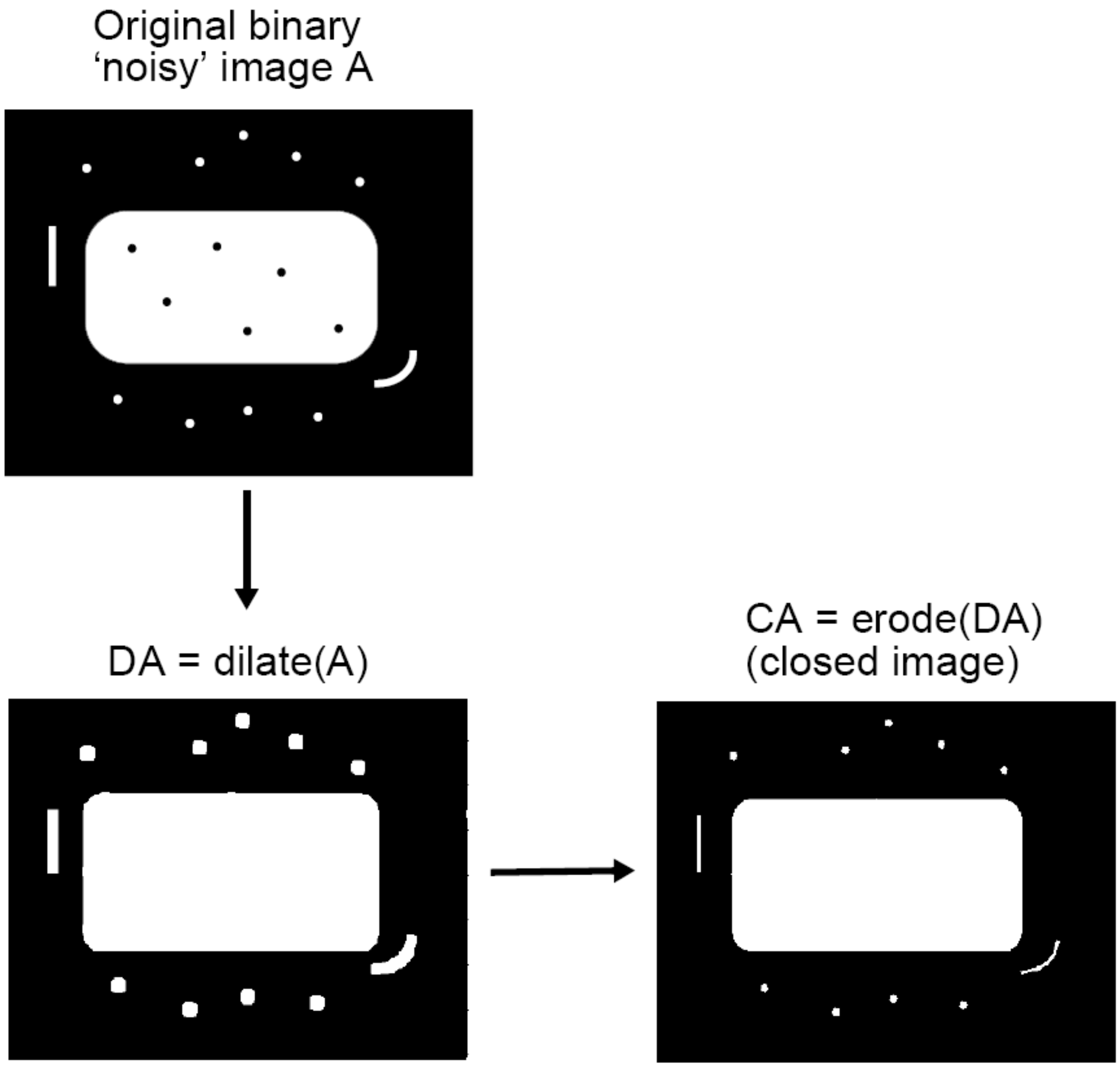
$$A \bullet B = (A \oplus B) \ominus B$$



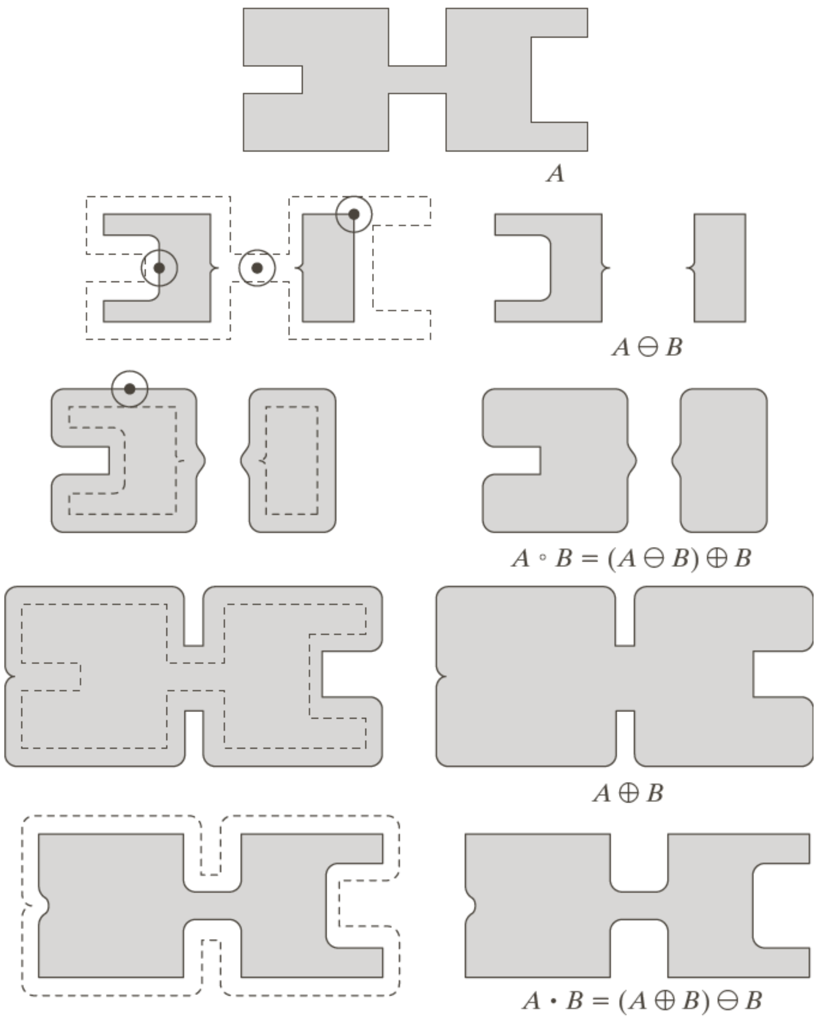
a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade A in (a) for clarity.

Binary Closing



Binary Opening and Closing

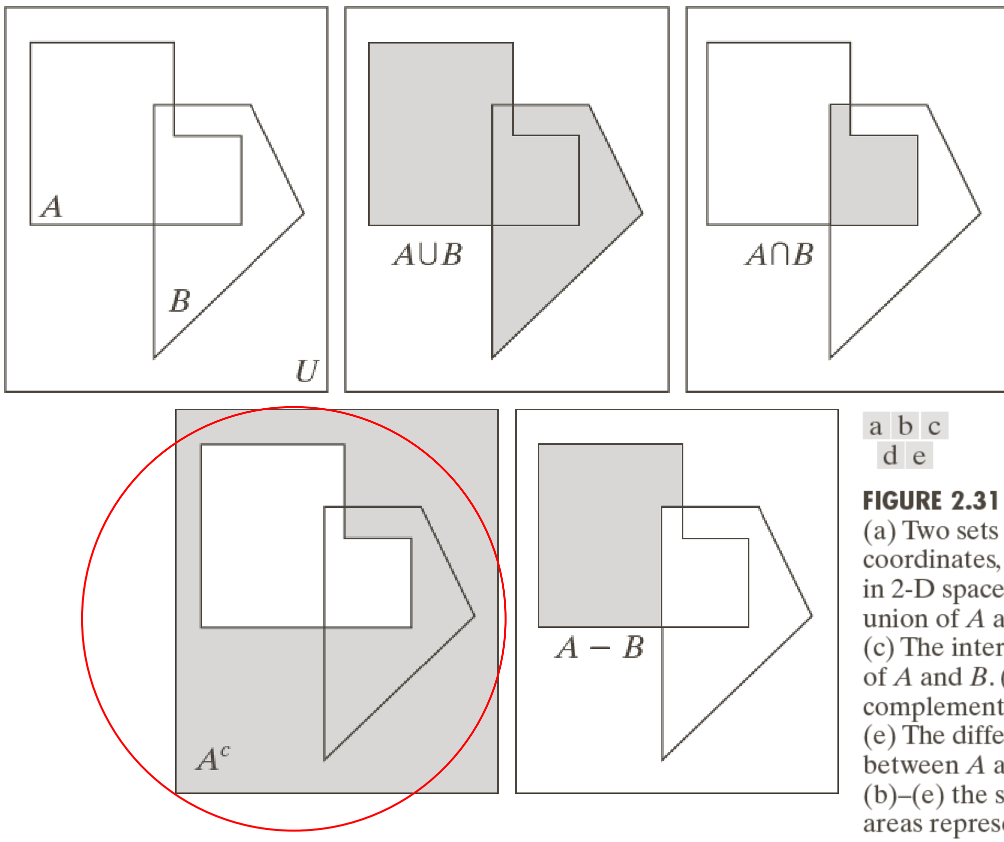


a	
b	c
d	e
f	g
h	i

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the structuring element.

Set Theory

Set Theory	Binary Logic
$A \cup B$	A OR B
$A \cap B$	A AND B
A^c	NOT A



a b c
d e

FIGURE 2.31
(a) Two sets of coordinates, A and B, in 2-D space. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B. In (b)–(e) the shaded areas represent the member of the set operation indicated.

Set Theory

- For a given set we define the reflection and the translation.

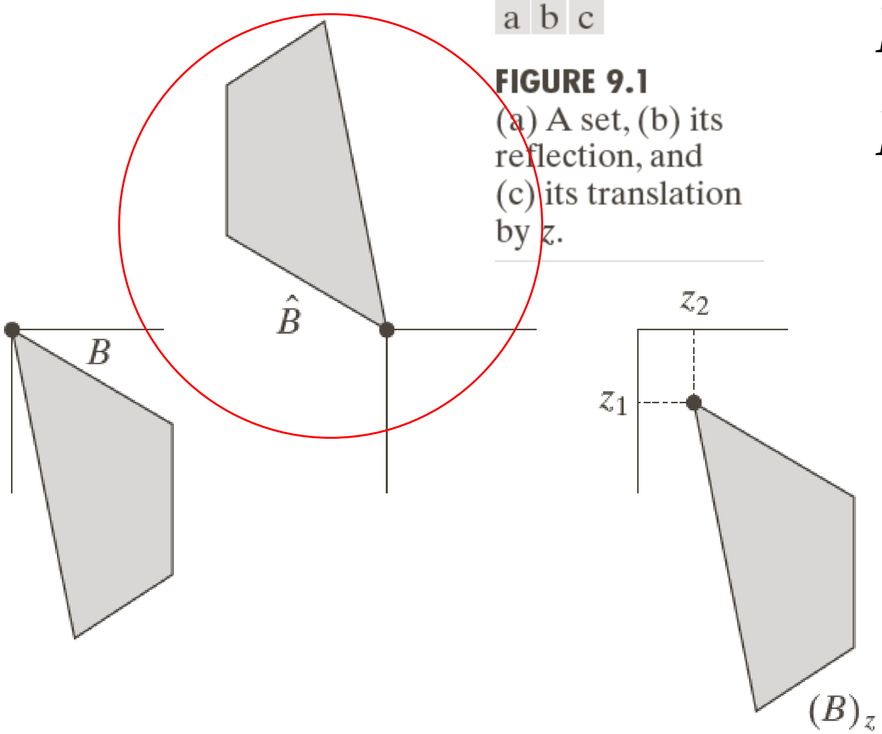


FIGURE 9.1 (a) A set, (b) its reflection, and (c) its translation by z.

$$\hat{B} = \{-b : b \in B\}$$

$$B_z = \{b + z : b \in B\},\ z = (z_1, z_2)$$

The reflection is the symmetric set with respect to the origin, all elements are scaled by -1.

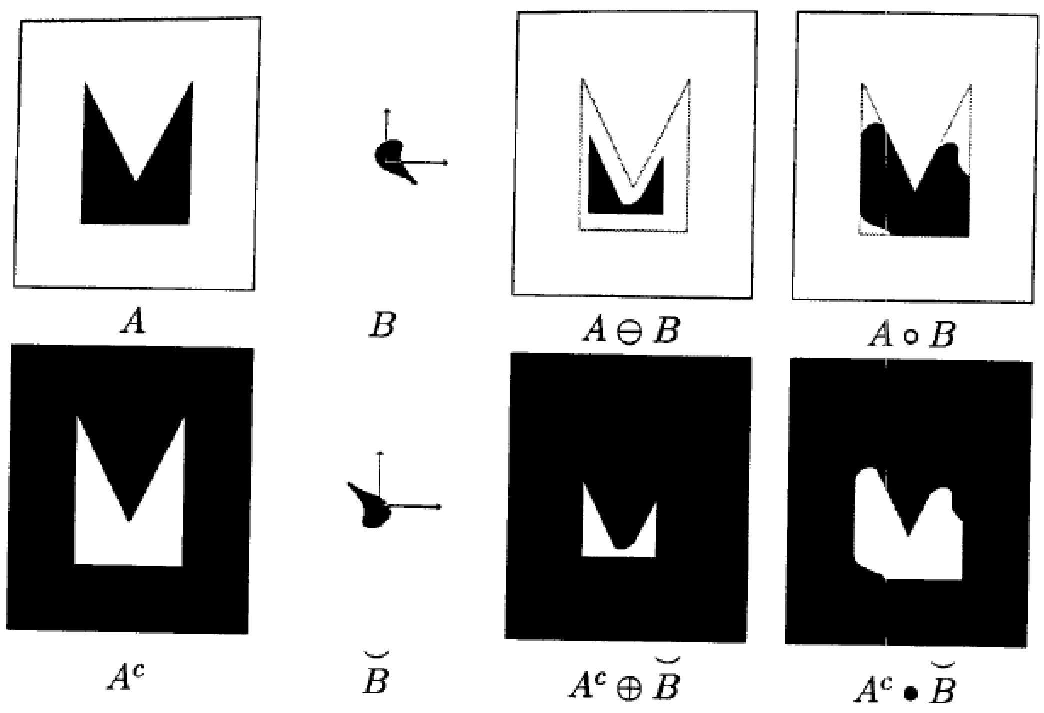
Properties of Opening and Closing

Duality with respect to complimentation

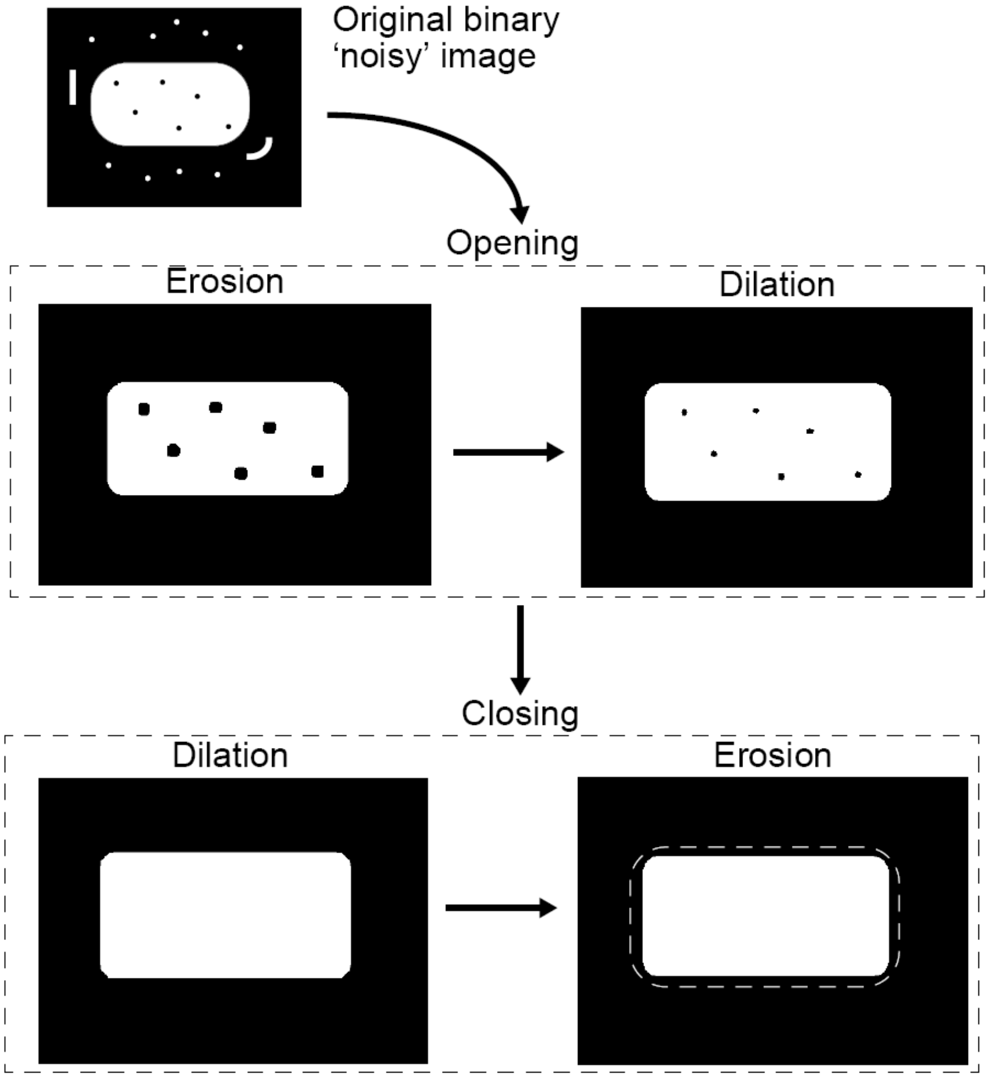
$$(A \bullet B)^c = (A^c \circ \hat{B})$$

$$(A \circ B)^c = (A^c \bullet \hat{B})$$

Erosion and dilation are also dual operations.



Noise Removal



- This opening-closing technique is the simplest form of a class of filters called alternating sequential filters.
  - Open-close filters
  - Close-open filters

- The alternating sequential filter typically alternates between open-close, or close-open operations using ever increasing structuring elements.

Noise Removal

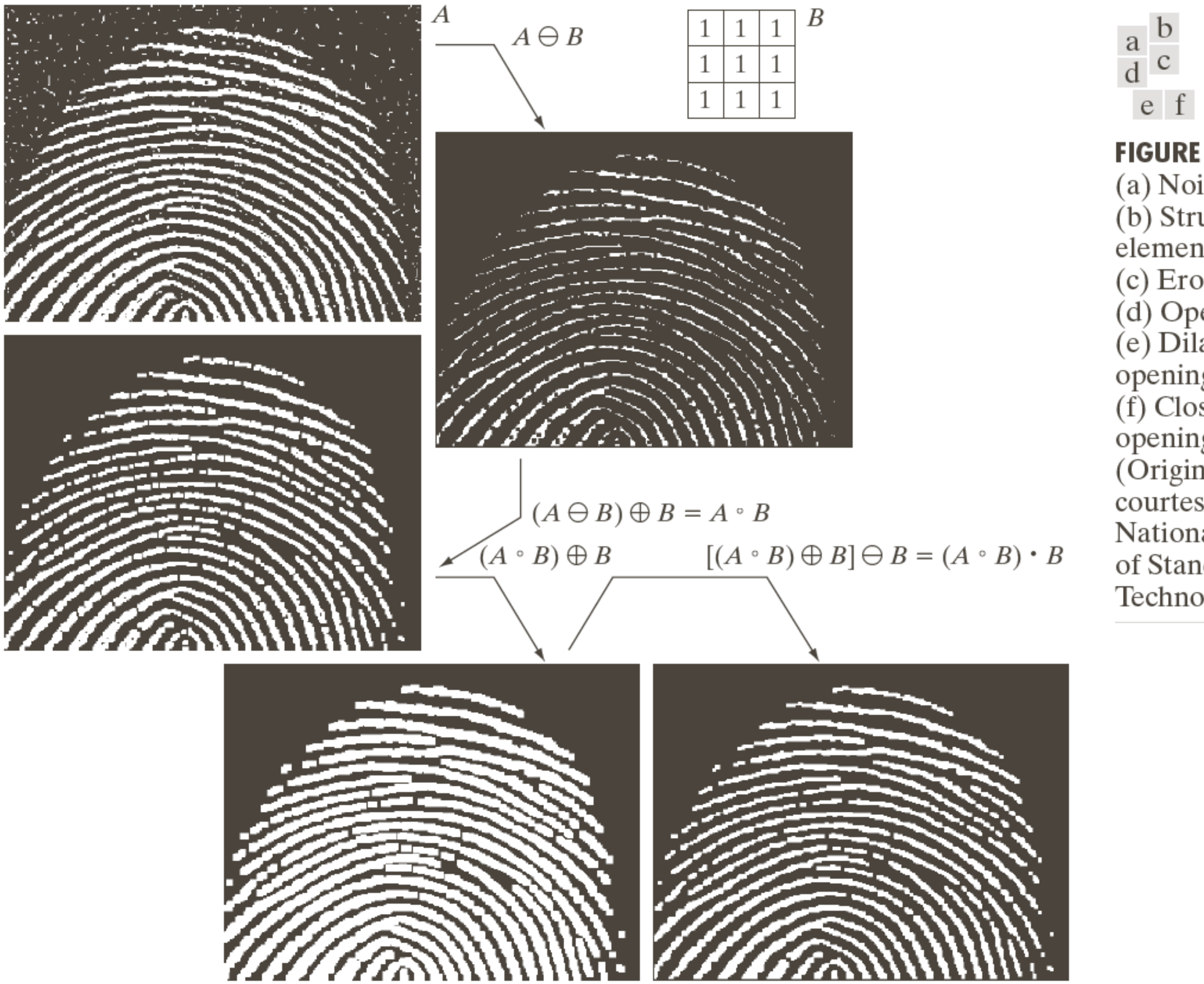


FIGURE 9.11
(a) Noisy image.
(b) Structuring element.
(c) Eroded image.
(d) Opening of A.
(e) Dilation of the opening.
(f) Closing of the opening.
(Original image courtesy of the National Institute of Standards and Technology.)

Morphological Boundary Detection

- Boundary (or edge) detection can be performed in a number of ways. Here we show techniques that detecting the internal and external boundaries of a set.

Internal boundary

$$\beta_B(A) = A - (A \ominus B)$$

External boundary

$$\gamma_B(A) = (A \oplus B) - A$$

Morphological Gradient

$$\gamma_B(A) = (A \oplus B) - (A \ominus B)$$

Morphological Laplacian

$$\begin{aligned}\omega_B(A) &= \gamma_B(A) - \beta_B(A) \\ &= (A \oplus B) - A - [A - (A \ominus B)] \\ &= (A \oplus B) + (A \ominus B) - 2A\end{aligned}$$

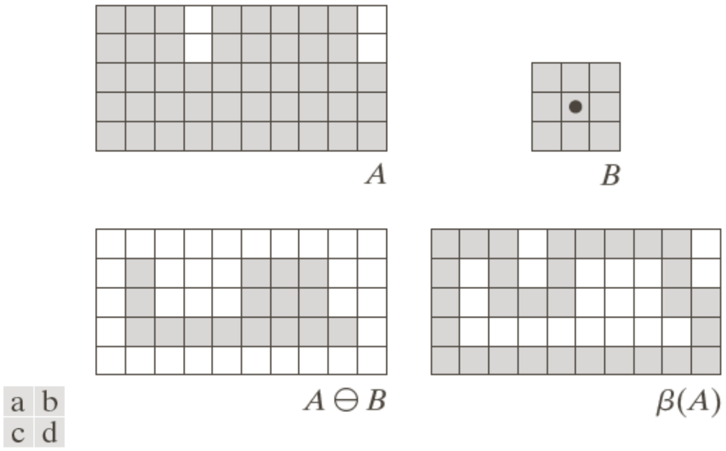


FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.

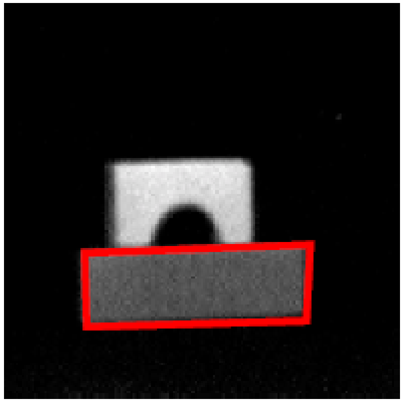
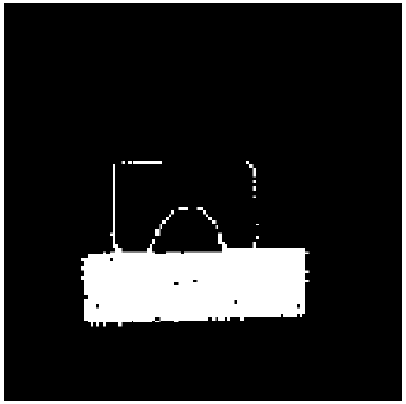
Morphological Boundary Detection

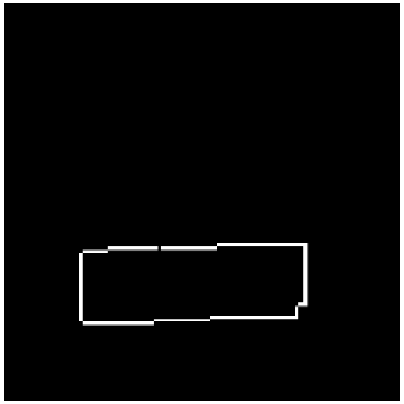
a b

FIGURE 9.14 (a) A simple binary image, with 1s represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Detection of the internal boundary

Morphological Boundary Detection



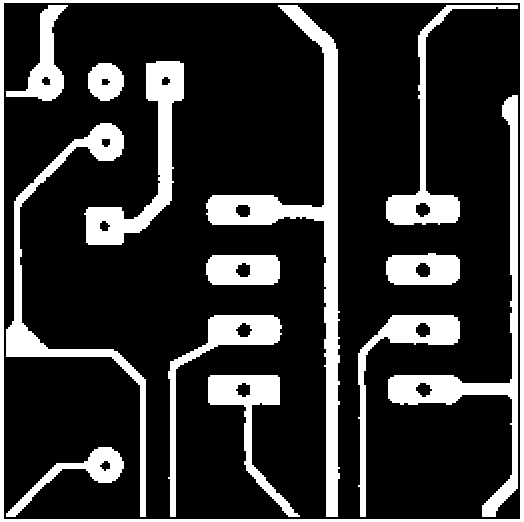
Original Grayscale Image ⇨ Thresholded ⇨ Open-Close Noise Removal ⇨ Inner Boundary

Introduction to More Advanced Topics

- Hole Filling
- Binary Segmentation
- Watershed Segmentation

Connected Components & Hole Filling

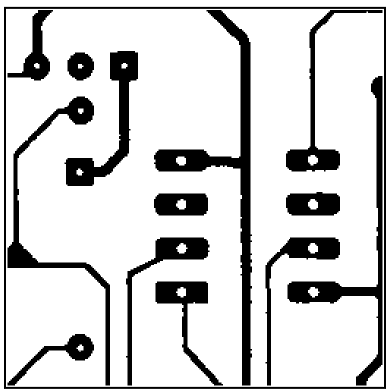
- We define a hole as a background region surrounded by a connected border of foreground pixels.
- We provide an algorithm for hole filling and detection based on connected components (DIP textbook 9.5.3 Connected Components, and not DIP 9.5.2 Hole Filling)
- We note that if a background region is a hole, then it is not connected to the boundary of the image. Therefore if we can detect the connected background of the image, the remainder will be the objects and the filled holes. Subtracting the original from this remainder will detect the holes.
- We apply this strategy to fill and detect the holes in the circuitboard image.



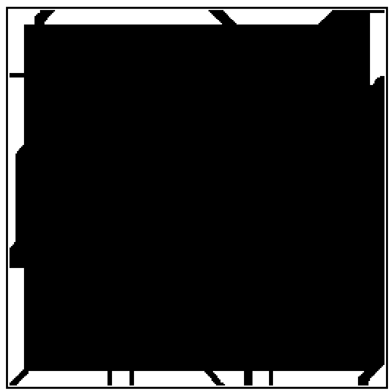
Connected Components & Hole Filling

- Image A is the inverse image of the circuitboard
- Image X_0 is a marker set, a black image with a one pixel white border.
- We repeatedly perform the following *conditional dilation* until $X_k = X_{k-1}$

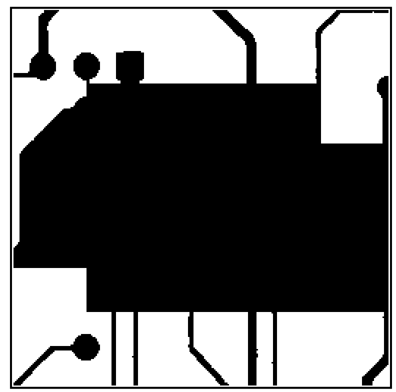
$$X_k = (X_{k-1} \oplus B) \cap A$$



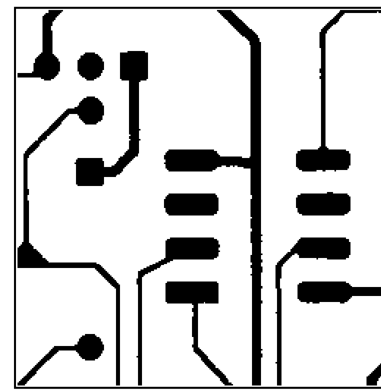
Inverted circuitboard



10 steps. X_{10}



50 steps. X_{50}



Complete

The `imfill` function in Matlab performs these flood filling conditional dilations.

Watershed Segmentation

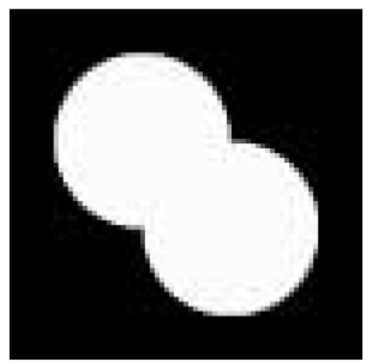
- We can use the watershed segmentation for distinguishing boundaries between overlapping objects in binary images.
- Using the distance transform we can turn a binary image into a greyscale surface. The distance transform of a binary image calculated the distance from every pixel to the nearest background pixel (black or 0).

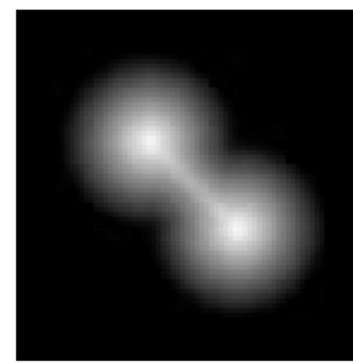
Binary image

0	0	1	1	1
0	0	1	1	1
1	1	1	1	1
1	1	1	1	1
1	0	0	0	1

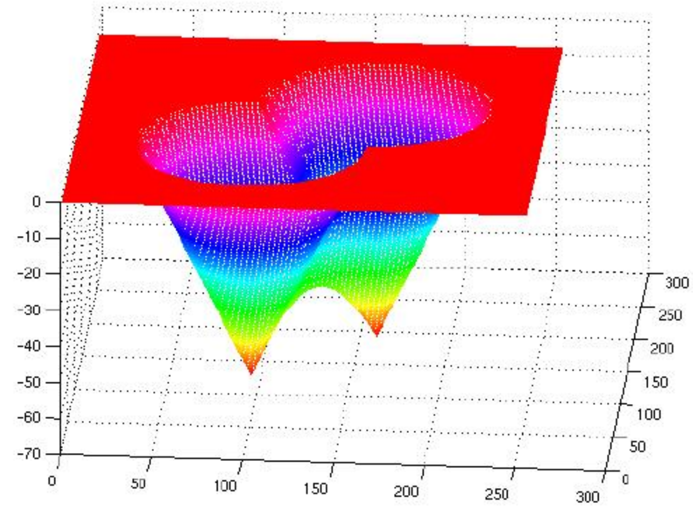
Distance transform

0	0	1	2	3
0	0	1	2	3
1	1	$\sqrt{2}$	2	$\sqrt{5}$
$\sqrt{2}$	1	1	1	$\sqrt{2}$
1	0	0	0	1





Watershed Segmentation



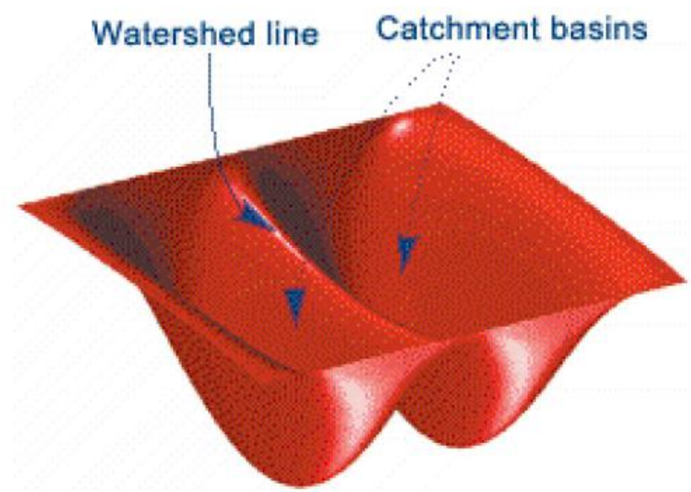
- The distance transform can be seen as a 3D surface, depicted here is the negative of the distance transform.
- The peaks in the distance transform are now the valleys in this 3D surface upon which we can apply the watershed segmentation.
- Watershed segmentation

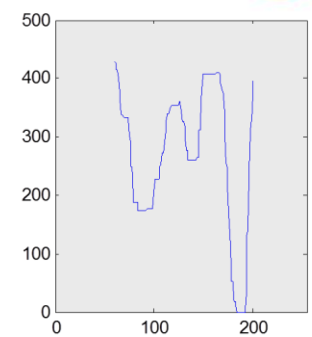
  This process creates 1 region from each local minima.

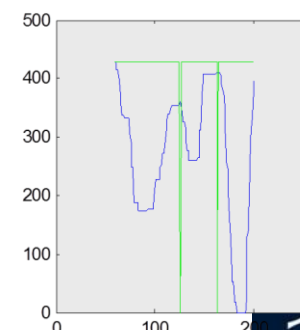
  Drill a hole in the surface at each local maxima

  Increase the watertable so that water flows out of the holes and begins flooding the valleys

  Create a boundary (a wall) where the water from two different valleys would merge.

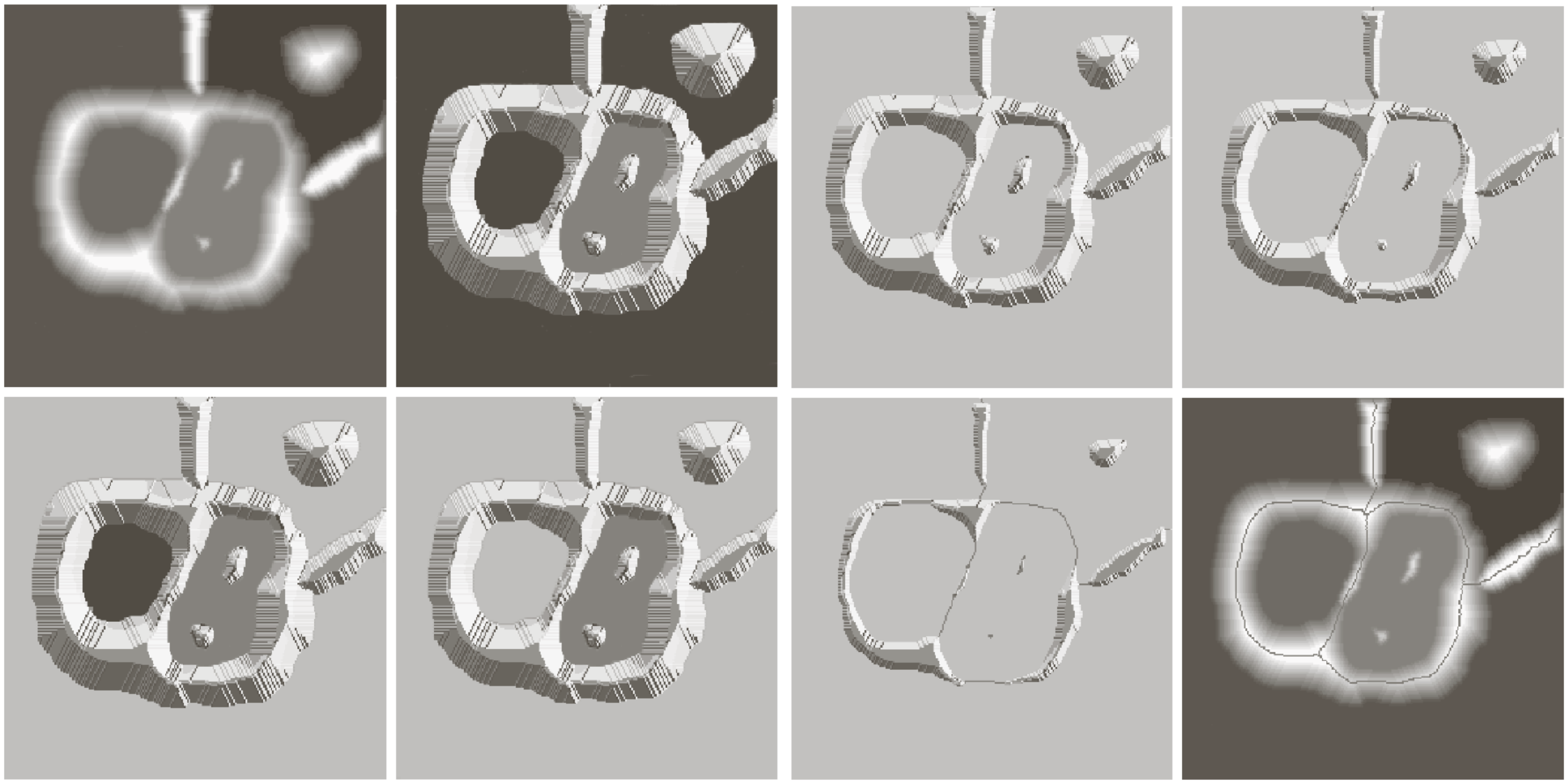






Note: Draw a 2D function and demonstrate the algorithm

Watershed Segmentation



e f
g h

FIGURE 10.54 *(Continued)* (e) Result of further flooding. (f) Beginning of merging of water from two catchment basins (a short dam was built between them). (g) Longer dams. (h) Final watershed (segmentation) lines. (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)

a b
c d

FIGURE 10.54 (a) Original image. (b) Topographic view. (c)–(d) Two stages of flooding.

Further reading: DIP 10.5 Segmentation using morphological waterhsheds

Summing Up

- Consider the following three questions;
  - What do I need to work on?
  - What have I learnt today?
  - What was the main point left unanswered today?

- Write your answers in the provided journal. Write the lecture number on top of the page.

End of Lecture

Not part of the course assessment

Properties of Opening and Closing

$$A \circ B \subset A$$

$$\text{if } C \subset D \text{ then } (C \circ B) \subset (D \circ B)$$

$$(A \circ B) \circ B = A \circ B$$

The opening of A by B removes all components of A that are smaller than B. There is no further effect by repeating the process. This property is called Idempotent.

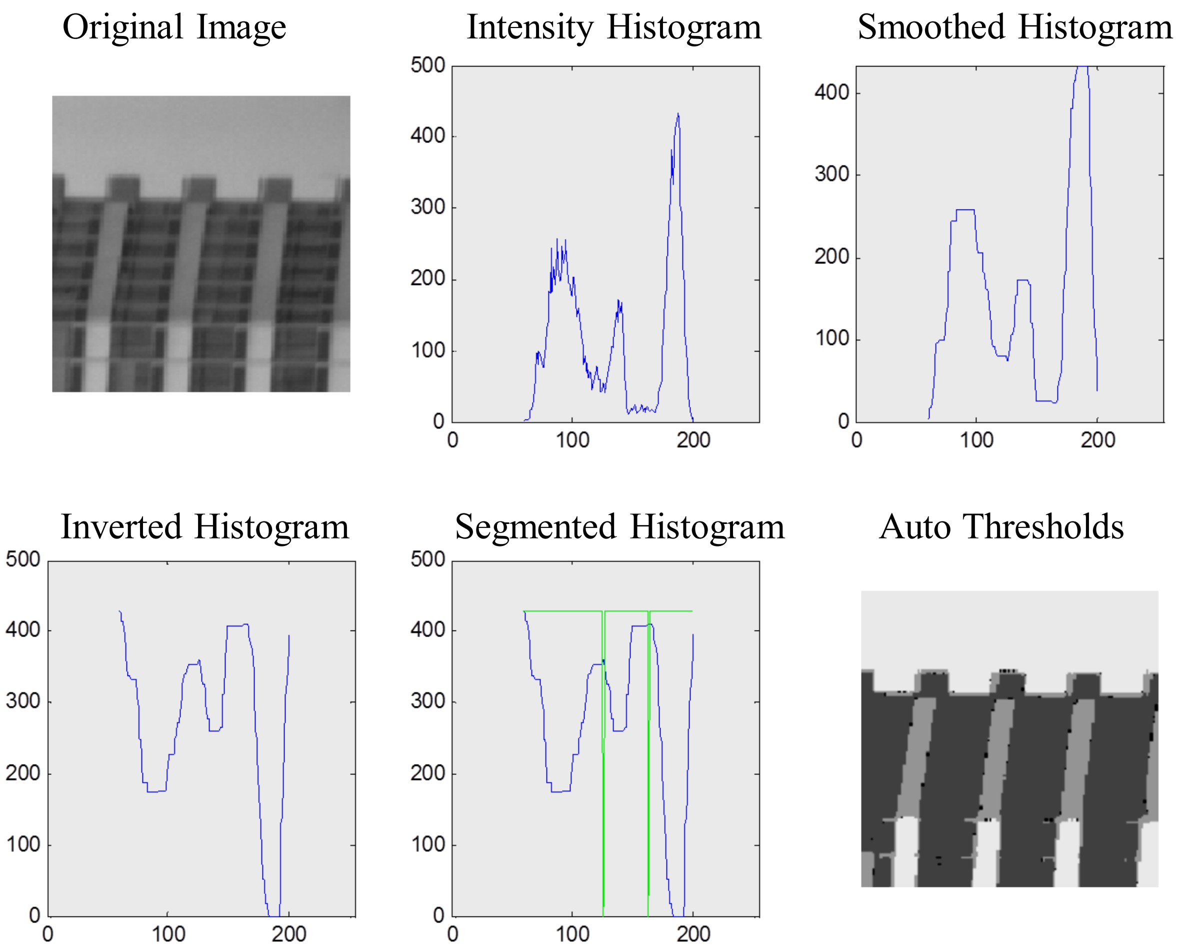
$$A \subset A \bullet B$$

$$\text{if } C \subset D \text{ then } (C \bullet B) \subset (D \bullet B)$$

$$(A \bullet B) \bullet B = A \bullet B$$

Otsu's Optimal Global Thresholding in DIP10.3.3 is more robust than this

Automatic Thresholding using watershed



Calculate the image histogram

Perform a smoothing to remove small local minima.

```
nlfilter(hist,[1 9],'max(x(:))');
```

There are better options than this maximum filter using morphological greyscale opening.

Invert the histogram to create a histogram of valleys instead of peaks

Perform watershed segmentation

This process creates 1 region from each local minima, creating boundaries between regions at the peaks.

Drill a hole in the ground at each local maxima

Increase the watertable so that water flows out of the holes and begins flooding the valleys

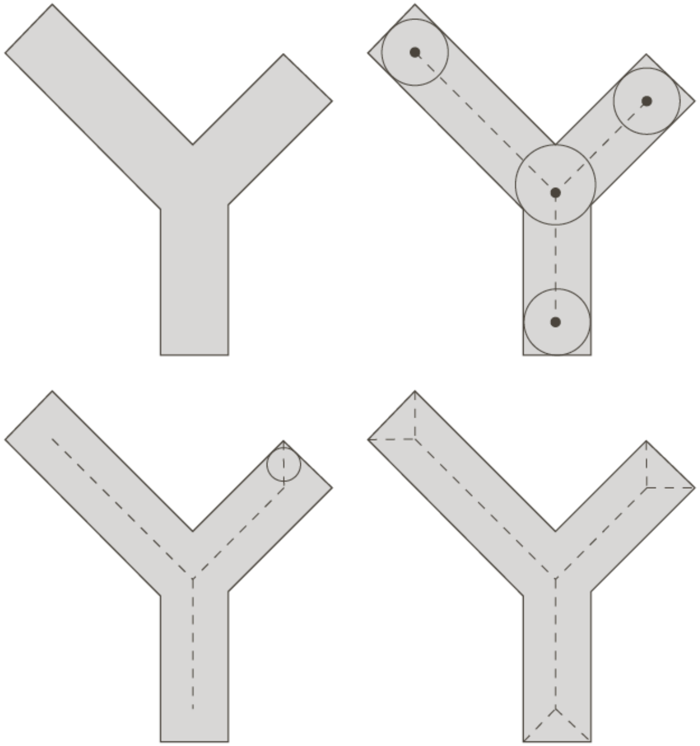
Create a boundary (a wall) where the water from two different valleys would merge.

The boundary walls identified by the watershed segmentation are our threshold boundaries.

Not part of the course assessment

Skeletonization an overview

- Through a combination of simple morphological operations we can extract the skeleton of an image.
- Given a point in the interior of a binary image there exists a largest disk within the image centred at this point.
- For a given disk, either there exists a larger disk containing this disk, OR it is the maximal disk at this point.
- The centres of all the maximal disks make the skeleton.



a b
c d

FIGURE 9.23 (a) Set A. (b) Various positions of maximum disks with centers on the skeleton of A. (c) Another maximum disk on a different segment of the skeleton of A. (d) Complete skeleton.