Advanced Image Processing and Image Segmentation Techniques – Segmentation

GRK-1564
Imaging
New Modalities

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Active Contour Model

Active Contour Model

Deformable Models

- "Deformable Model – a group of computer algorithms and techniques widely used in computer vision today. They all share the common characteristic that they model the variability of a certain class of objects."
- "Segmentation – the shape, represented as curve or surface, is deformed in order to match a specific example in the object class."
  - Active contours
  - Deformable surfaces
  - Deformable templates
  - Modeling of texture variation

Active Contour Model

Definition

"The active contour model, or **snake**, is defined as an energy minimizing spline – the snake's energy depends on its shape and location within the image."

- parametric
- geometric
- geodesic

Active Contour Model

Idea

The curve changes its shape under the influence of applied internal and external forces.

Applications:

- modeling
- segmentation
- objects tracking
- face recognition

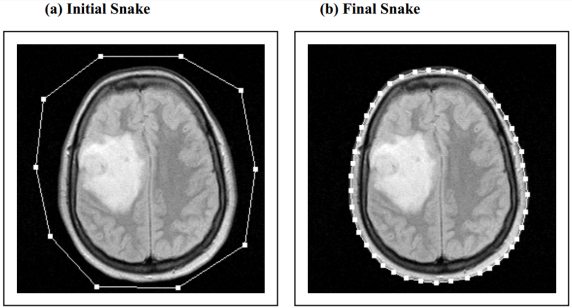
Active Contour Model

Definitions

Contour:

- parametric curve
- points

Principle of action:



http : //www.computing.edu.au/ jim/thesis/ivins02.pdf

Active Contour Model

Contour

Parametric representation of contour $\Psi(s) : \mathbb{R}^2 \rightarrow \mathbb{R}$

$$\Psi(s) = (x(s), y(s)) \quad s \in [0, 1]$$

Assumptions:

Contour is closed curve

$$\Psi(0) = \Psi(1)$$

Contour changes in time

$$\Psi(s, t) = (x(s, t), y(s, t)) \quad s \in [0, 1], t = \mathbb{R}_+ \cup \{0\}$$

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Goal:

Minimization of curve energy E_{snake} in following iterations – local minima of this energy then correspond to desired image properties. Time and way to find the solution depend on:

- model parameters - external energy
- starting conditions
- internal energy

Local method – in each iteration, only the nearest surrounding of the curve points is analyzed.

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Model energy:

$$E_{snake} = \int_0^1 \left(E_{snake}(\Psi(s,t))\right) ds =$$

$$= \int_0^1 \left(E_{int}(\Psi(s,t)) + E_{ext}(\Psi(s,t)) + E_{con}(\Psi(s,t))\right) ds$$

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Model energy:

- The energy functional which is minimized is a weighted combination of internal and external force, and in some cases external constraints.
- The internal forces emanate from the shape of the snake, while the external forces come from the image and/or from higher level image understanding processes.

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Internal energy:

Describes the curve:

$$E_{int}(\Psi(s,t)) = \frac{1}{2}\left(\alpha\left|\frac{\partial\Psi}{\partial s}\right|^2 + \beta\left|\frac{\partial^2\Psi}{\partial s^2}\right|^2\right)$$

α – tension

β – rigidity

$\frac{\partial\Psi}{\partial s}$ – determines curve elasticity, it treats the curve as a membrane – it causes, that the curve does not stretch

$\frac{\partial^2\Psi}{\partial s^2}$ – determines the stiffness of the curve

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External energy:

Describes the image content – causes the movement of snake towards the regions of specific parameters:

$$E_{image} = \omega_{line} E_{line} + \omega_{edge} E_{edge} + \omega_{term} E_{term}$$

an exemplary weighted combination of three different functionals, which attracts the snake to lines, edges, and terminations - corners.

External energy:

Describes the image content – causes the movement of snake towards the regions of specific parameters:

- $E_{ext} = I(x, y)$ - the snake moves towards dark or bright regions - lines
- $E_{ext} = -|\nabla I(x, y)|^2$ - the snake moves towards the regions with high gradient - edges
- $E_{ext} = \frac{C_{yy}C_x^2 - 2C_{xy}^2 C_x C_Y + C_{xx}C_y^2}{(C_x^2 + C_y^2)^{\frac{3}{2}}}$ - corners where:

  $C_x = \frac{\partial I}{\partial x},\ C_y = \frac{\partial I}{\partial y},\ C_{xx} = \frac{\partial^2 I}{\partial x^2},\ C_{yy} = \frac{\partial^2 I}{\partial y^2},\ C_{xy} = \frac{\partial^2 I}{\partial x \partial y}$

Mostly the external energy is estimated for the image smoothed with Gaussian filter

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External constraints:

It comes from external constraints imposed either by a user or some other higher level process which may force the snake toward or away from particular features:

- Spring energy: spring between two points $\mathbf{x}_1$ and $\mathbf{x}_2$, one located on the snake and the second located in image

$$E_{term} = -k|\mathbf{x}_1 - \mathbf{x}_2|$$

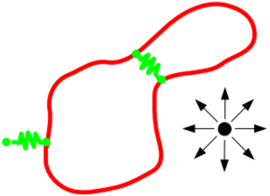
- Volcano energy: local repulsing force pushing the snake away form some specific image areas (local minimum):

$$E_{term} = min(peak, \frac{1}{r})$$

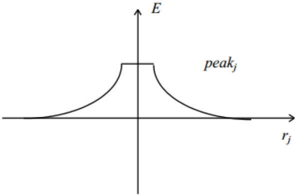
where r - radius of the volcano - distance between point $\mathbf{c}$ and the vertex

Active Contour Model

External constraints:



Spring Energy



Volcano Energy

http : //aragorn.pb.bialystok.pl/ boldak/DIP/CPO — W08 — v01 — 50pr.pdf

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According to the calculus of variations, the curve $\Psi(s,t)$ minimizing the energy E_{snake} zeroes out the Euler-Lagrange equation (Euler equation of motion):

$$\frac{\partial}{\partial s}\left(\alpha\frac{\partial\Psi}{\partial s}\right)-\frac{\partial^2}{\partial s^2}\left(\beta\frac{\partial^2\Psi}{\partial s^2}\right)-\nabla E_{ext}=0$$

Let:

$$-\nabla E_{ext}=F_{ext}=\begin{bmatrix} f_x \\ f_y \end{bmatrix}$$

$$f_x=\frac{\partial E_{ext}}{\partial x},\quad f_y=\frac{\partial E_{ext}}{\partial y}$$

then

$$\alpha\left(\frac{\partial^2\Psi}{\partial s^2}\right)-\beta\left(\frac{\partial^4\Psi}{\partial s^4}\right)+F_{ext}=0$$

Contour discretization

Discrete contour:

$$\underline{s}_i^\tau = [x_i^\tau, y_i^\tau]$$

$\tau = 0, 1, 2, \ldots, T$ – iteration
$i = 0, 1, 2, \ldots, (N-1)$ – curve points
Since:

$$\underline{s}_0^\tau = \underline{s}_{N-1}^\tau,$$

$$\underline{\Psi}(\underline{s}^\tau) = [X^\tau, Y^\tau]$$

$X^\tau = [x_0^\tau, x_1^\tau, \ldots, x_{N-1}^\tau]$ – vector of abscissas
$Y^\tau = [y_0^\tau, y_1^\tau, \ldots, y_{N-1}^\tau]$ – vector of ordinates

Contour discretization

$$\frac{\partial^2 \underline{\Psi}}{\partial x_i^{\tau 2}} = \frac{x_{i-1}^{\tau} - 2x_i^{\tau} + x_{i+1}^{\tau}}{2h_x^2}$$

$$\frac{\partial^2 \underline{\Psi}}{\partial y_i^{\tau 2}} = \frac{y_{i-1}^{\tau} - 2y_i^{\tau} + y_{i+1}^{\tau}}{2h_y^2}$$

$$\frac{\partial^4 \underline{\Psi}}{\partial x_i^{\tau 4}} = \frac{x_{i-2}^{\tau} - 4x_{i-1}^{\tau} + 6x_i^{\tau} - 4x_{i+1}^{\tau} + x_{i+2}^{\tau}}{2h_x^4}$$

$$\frac{\partial^4 \underline{\Psi}}{\partial y_i^{\tau 4}} = \frac{y_{i-2}^{\tau} - 4y_{i-1}^{\tau} + 6y_i^{\tau} - 4y_{i+1}^{\tau} + y_{i+2}^{\tau}}{2h_y^4}$$

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$$\alpha\left(\frac{x_{i-1}^{\tau}-2x_i^{\tau}+x_{i+1}^{\tau}}{2h_x^2}\right)-\beta\left(\frac{x_{i-2}^{\tau}-4x_{i-1}^{\tau}+6x_i^{\tau}-4x_{i+1}^{\tau}+x_{i+2}^{\tau}}{2h_x^4}\right)+ \\ +f_x(x_i^{\tau-1},y_i^{\tau-1})=0$$

$$\alpha\left(\frac{y_{i-1}^{\tau}-2y_i^{\tau}+y_{i+1}^{\tau}}{2h_y^2}\right)-\beta\left(\frac{y_{i-2}^{\tau}-4y_{i-1}^{\tau}+6y_i^{\tau}-4y_{i+1}^{\tau}+y_{i+2}^{\tau}}{2h_y^4}\right)+ \\ +f_y(x_i^{\tau-1},y_i^{\tau-1})=0$$

After ordering

$$-\beta x_{i-2}^{\tau} + (\alpha + 4\beta)x_{i-1}^{\tau} + (-2\alpha - 6\beta)x_i^{\tau} + (\alpha + 4\beta)x_{i+1}^{\tau} - \\ -\beta x_{i+2}^{\tau} + 2f_x(x_i^{\tau-1}, y_i^{\tau-1}) = 0$$

$$-\beta y_{i-2}^{\tau} + (\alpha + 4\beta)y_{i-1}^{\tau} + (-2\alpha - 6\beta)y_i^{\tau} + (\alpha + 4\beta)y_{i+1}^{\tau} - \\ -\beta y_{i+2}^{\tau} + 2f_y(x_i^{\tau-1}, y_i^{\tau-1}) = 0$$

Active Contour Model

Matrix equation

$$AX^{\tau} + 2f_x(X^{\tau-1}, Y^{\tau-1}) = 0$$

$$AY^{\tau} + 2f_y(X^{\tau-1}, Y^{\tau-1}) = 0$$

where A is symmetric and pentadiagonal matrix useful when solving the discrete equations of motion of the snake:

$$\begin{bmatrix}
-2\alpha - 6\beta & \alpha + 4\beta & -\beta & 0 & \cdots & -\beta & \alpha + 4\beta \\
\alpha + 4\beta & -2\alpha - 6\beta & \alpha + 4\beta & -\beta & \cdots & 0 & -\beta \\
-\beta & \alpha + 4\beta & -2\alpha - 6\beta & \alpha + 4\beta & \cdots & 0 & 0 \\
0 & -\beta & \alpha + 4\beta & -2\alpha - 6\beta & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
-\beta & 0 & 0 & 0 & \ddots & -2\alpha - 6\beta & \alpha + 4\beta \\
\alpha + 4\beta & -\beta & 0 & 0 & \cdots & \alpha + 4\beta & -2\alpha - 6\beta
\end{bmatrix}$$

Replacing right sides of the equations with partial time derivatives of curve – for the big enough time, the energy of curve will get the extreme:

$$AX^{\tau} + 2f_x(X^{\tau-1}, Y^{\tau-1}) = \frac{\partial \Psi}{\partial t}$$

$$AY^{\tau} + 2f_y(X^{\tau-1}, Y^{\tau-1}) = \frac{\partial \Psi}{\partial t}$$

$$\frac{\partial X^{\tau}}{\partial t} = \frac{X^{\tau} - X^{\tau-1}}{\gamma}$$

$$\frac{\partial Y^{\tau}}{\partial t} = \frac{Y^{\tau} - Y^{\tau-1}}{\gamma}$$

After placing:

$$\frac{X^{\tau} - X^{\tau-1}}{\gamma} = AX^{\tau} + 2f_x(X^{\tau-1}, Y^{\tau-1})$$

$$\frac{Y^{\tau} - Y^{\tau-1}}{\gamma} = AY^{\tau} + 2f_y(X^{\tau-1}, Y^{\tau-1})$$

$$X^{\tau} = (I - \gamma A)^{-1}(X^{\tau-1} + 2\gamma f_x(X^{\tau-1}, Y^{\tau-1}))$$

$$Y^{\tau} = (I - \gamma A)^{-1}(Y^{\tau-1} + 2\gamma f_y(X^{\tau-1}, Y^{\tau-1}))$$

Algorithm:

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====== Algorithm 1. Snake Movement =========
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1: $\tau \longleftarrow 1$;

2: Initialize the curve $\underline{\Psi}(\underline{s}^{\tau})$ with vectors X^{τ}, Y^{τ} ;

3: Calculate the elements of matrix A ;

4: Repeat

5: Calculate new vectors X^{τ}, Y^{τ} ;

6: $\tau \longleftarrow \tau + 1$;

7: Estimate the error $\varepsilon_{\Psi}^{\tau}$;

8: until $(\varepsilon_{\Psi}^{\tau} \leqslant \varepsilon_{\Psi})$

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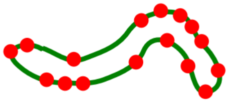
Active Contour Model

Stop conditions

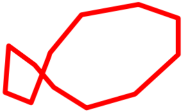
- the coordinates of curve points do not change any more
- the energy changes are small enough
- the number of performed iterations is equal to the T set at the beginning

Drawbacks

- Big dependence of parameters α i β
- Inhomogeneous gradient vector field - slow convergence of curve, zeros gradient vector field causes stopping of curve movement - it causes problems with segmenting concave shapes
- Huge dependence on starting curve shape
- Dependence of energy of scale
- Problem of keeping the contour form - loops formation



Irregular distribution of points



Loops formation

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Drawbacks

- Shrinking of snake

Balloon/pressure force

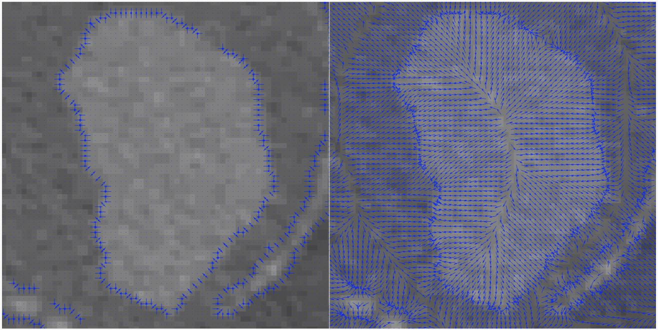
The balloon force is applied on the contour points in the normal direction of the contour and ensures that the contour is expanded or contracted with a constant speed. This is similar to the effect of α:

$$F_{balloon}(s) = kn(\Psi(s))$$

where $n(\Psi(s))$ - contour normal

Gradient Vector Flow

The GVF forces tend to extend very far away from the - the edge influence is spread



Gradient field

$$\nabla f = \nabla E_{ext}$$

- a directional change in the intensity - local in the relation to the edges

GVF

$$V(x,y) = (u(x,y), v(x,y))$$

- extend the gradient vector field (along the vectors of the field) far away from the object, so that that snakes can find objects that are quite far away from the snake's, where it was not previously defined, minimizing the energy functional:

$$E_{GVF} = \int\int \mu(u_x^2+u_y^2+v_x^2+v_y^2)+(f_x^2+f_y^2)((u-f_x)^2-(v-f_y)^2)dxdy$$

GVF

- when $(f_x^2 + f_y^2)$ is small, the energy is dominated by sum of the squares of the partial derivatives of the vector field, yielding a slowly varying field.
- when $(f_x^2 + f_y^2)$ is large, the second term dominates the integrand, and is minimized by setting. This produces the desired effect of keeping nearly equal to the gradient of the edge map when it is large, but forcing the field to be slowly-varying in homogeneous regions.
- μ - a regularization parameter governing the tradeoff between the first term and the second term in the integrand. This parameter should be set according to the amount of noise present in the image (more noise, increase).

Active Contour Model

GVF

According to the calculus of variations (Euler-Lagrange equation) the solution minimizing the energy E_{GVF}:

$$\mu\nabla^2 u - (u - f_x)(f_x^2 + f_y^2) = 0$$
$$\mu\nabla^2 v - (v - f_y)(f_x^2 + f_y^2) = 0$$

where ∇^2 - Laplacian operator
Treating it as functions of time:

$$\begin{aligned} u_t(x,y,t) = \mu\nabla^2 u(x,y,t) - (f_x(x,y)^2 + f_y(x,y)^2)u(x,y,t) + \\ + (f_x(x,y)^2 + f_y(x,y)^2)f_x(x,y) \\ v_t(x,y,t) = \mu\nabla^2 v(x,y,t) - (f_x(x,y)^2 + f_y(x,y)^2)v(x,y,t) + \\ (f_x(x,y)^2 + f_y(x,y)^2)f_x(x,y) \end{aligned}$$

GVF

Assuming the derivative with respect to the time:

$$V_t(x, y) = \frac{1}{\triangle\ t}(V(x, y, t+1) - V(x, y, t)),$$

and the Laplacian operator:

$$\bigtriangledown^2 V(x, y) = \tfrac{1}{\triangle x \triangle y}(V(x+1, y) +$$
$$+V(x-1, y) + V(x, y+1) + V(x, y-1) - 4V(x, y)),$$

GVF

in the following iterations GVF evolves according to:

$$V(x,y,t+1) = (1 - b(x,y) \triangle t)V(x,y,t) + \\ +r(V(x+1,y) + V(x-1,y) + V(x,y+1) \\ +V(x,y-1) - 4V(x,y)) + \begin{bmatrix} c^1(x,y) \\ c^2(x,y) \end{bmatrix}$$

where

$$b(x,y) = f_x^2(x,y) + f_y^2(x,y)$$
$$c^1(x,y) = b(x,y)f_x(x,y)$$
$$c^2(x,y) = b(x,y)f_y(x,y)$$
$$r = \frac{\mu \triangle t}{\triangle x \triangle y}$$
$$\triangle t \leq \frac{\triangle x \triangle y}{4\mu}$$

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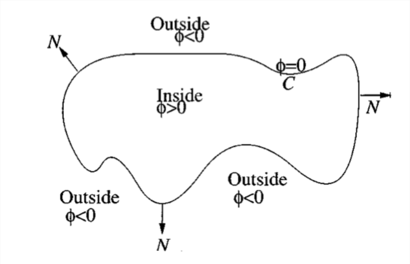
Active Contour Model

Chan & Vese - Active Contours Without Edges

Minimizing energy:

$$
\begin{aligned}
F(c_1, c_2, C) = \mu Length(C) + \nu Area(inside(C)) + \\
+\lambda_1 \int_{inside(C)} |I(x,y) - c_1|^2 dxdy + \\
+\lambda_2 \int_{outside(C)} |I(x,y) - c_2|^2 dxdy +
\end{aligned}
$$



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