Numerical Methods for Visual Computing
M. Möller, J. Geiping, University of Siegen
Winter Semester 18/19

Additional Information about Sheet 2 - Exercises 4

October 16, 2018

Exercise 1 (4 points)**.** Download the file *exerciseSheet2Code.zip* from the course homepage. You find a .mat file containing two images as well as two lists of points $p^1 \in \mathbb{R}^{2\times 190}$ and $p^2 \in \mathbb{R}^{2\times 190}$. Your goal is to estimate the vector $\vec{x} = (x_1, x_2, \dots, x_6)$ that determines the best affine transform between p^1, p^2, i.e.

$$\vec{x} = \arg\min_{\vec{x}} \sum_{i=1}^{190} \left\| \begin{pmatrix} x_1 & x_3 \\ x_2 & x_4 \end{pmatrix} \begin{pmatrix} p^1_{1,i} \\ p^1_{2,i} \end{pmatrix} + \begin{pmatrix} x_5 \\ x_6 \end{pmatrix} - \begin{pmatrix} p^2_{1,i} \\ p^2_{2,i} \end{pmatrix} \right\|^2 . \tag{1}$$

Hint: First construct a matrix P_i such that

$$P_i \vec{x} = \begin{pmatrix} x_1 & x_3 \\ x_2 & x_4 \end{pmatrix} \begin{pmatrix} p^1_{1,i} \\ p^1_{2,i} \end{pmatrix} + \begin{pmatrix} x_5 \\ x_6 \end{pmatrix}$$

Then stack all P_i to bring the problem into a more standard form.

Once you found the parameter vector $\vec{x}$ you can visualize the image stitching result you obtain with your parameters.

Since some of you are working in different programming languages, we also uploaded the data in a .cvs format. The image stitching part is just to have a nice visualization and application of the more technical underlying task. In fact, you can do the exercise without any images.

In this case, it boils down to the registration of two point clouds via an affine transformation as illustrated in figure 1: Try to find the 6 parameters $x_1, \dots, x_6$ of an affine linear transformation such that sum of squared distances between the 190 points in pointsInImage1.cvs and in pointsInImage2.cvs ist minimized. This is exactly what equation (1) expressed. Note that

$$\begin{aligned} \begin{pmatrix} x_1 & x_3 \end{pmatrix} \begin{pmatrix} p^1_{1,i} \\ p^1_{2,i} \end{pmatrix} + x_5 - p^2_{1,i} &= \begin{pmatrix} p^1_{1,i} & p^1_{2,i} & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_3 \\ x_5 \end{pmatrix} - p^2_{1,i} \\ &= \begin{pmatrix} p^1_{1,i} & 0 & p^1_{2,i} & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} - p^2_{1,i} \end{aligned}$$

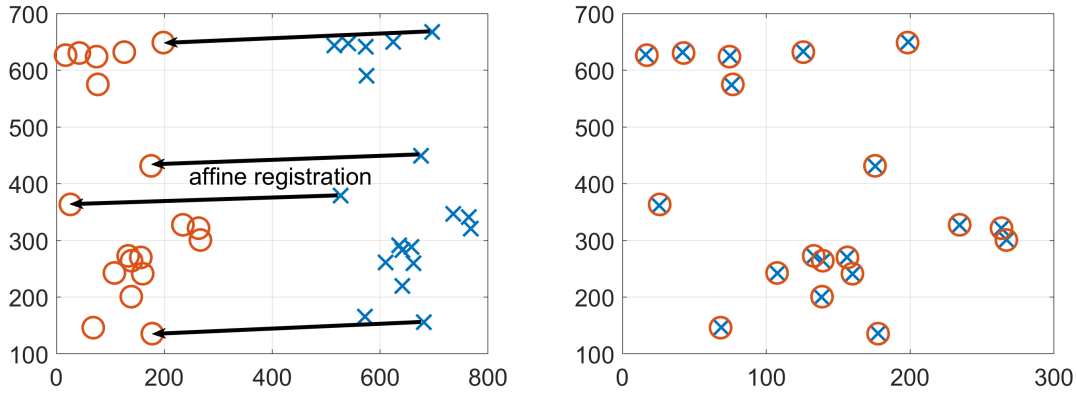

Figure 1: Registering two point clouds: Given two point clouds with known point-to-point correspondences (left) the goal of this exercise is to compute the optimal affine linear transformation to align the points (as successfully done on the right).