Being averaged for smaller α

Since G is averaged with respect to α there exists a nonexpansive operator R such that $G = \alpha I + (1-\alpha)R$. We find

$$\begin{aligned} G &= \alpha I + (1-\alpha)R \\ &= \tilde{\alpha} I + (\alpha - \tilde{\alpha})I + (1-\alpha)R \\ &= \tilde{\alpha} I + (1-\tilde{\alpha}) \underbrace{\left(\frac{\alpha-\tilde{\alpha}}{1-\tilde{\alpha}} I + \frac{1-\alpha}{1-\tilde{\alpha}} R \right)}_{=:\tilde{R}}. \end{aligned}$$

And $\tilde{R}$ is still nonexpansive because

$$\begin{aligned} \|\tilde{R}(u) - \tilde{R}(v)\| &\leq \frac{\alpha-\tilde{\alpha}}{1-\tilde{\alpha}} \|u-v\| + \frac{1-\alpha}{1-\tilde{\alpha}} \|R(u) - R(v)\| \\ &\leq \frac{\alpha-\tilde{\alpha}}{1-\tilde{\alpha}} \|u-v\| + \frac{1-\alpha}{1-\tilde{\alpha}} \|u-v\| \\ &= \|u-v\|. \end{aligned}$$

Composition of averaged operators

Let $G_1 = \alpha_1 I + (1-\alpha_1)R_1$ and $G_2 = \alpha_2 I + (1-\alpha_2)R_2$ for nonexpansive operators R_1 and R_2. Then

$$\begin{aligned} G_2(G_1)(u) &= \alpha_2 G_1(u) + (1-\alpha_2)R_2(G_1(u)) \\ &= \alpha_1\alpha_2 u + \alpha_2(1-\alpha_1)R_1(u) + (1-\alpha_2)R_2(G_1(u)) \\ &= \alpha_1\alpha_2 u + (1-\alpha_1\alpha_2)\left(\frac{\alpha_2(1-\alpha_1)}{1-\alpha_1\alpha_2} R_1(u) + \frac{(1-\alpha_2)}{1-\alpha_1\alpha_2} R_2(G_1(u)) \right). \end{aligned}$$

Since the concatenation of nonexpansive operators is nonexpansive, and convex combinations of nonexpansive operators are nonexpansive, we conclude that $G_2 \circ G_1$ is averaged.

Firmly nonexpansive operators are averaged operators with $\alpha = 1/2$

First, let G be averaged with $\alpha = 1/2$, i.e. let $G = \frac{1}{2}I + \frac{1}{2}R$ for some nonexpansive operator R, i.e. let $2G - I$ be nonexpansive. We find

$$\begin{aligned} \|u-v\|_2^2 \geq \|R(u) - R(v)\|^2 &= \|2G(u) - 2G(v) - (u-v)\|^2 \\ &= 4\|G(u) - G(v)\|^2 - 4\langle G(u) - G(v), u - v\rangle + \|u-v\|^2, \end{aligned}$$

which implies $\langle G(u) - G(v), u - v\rangle \geq \|G(u) - G(v)\|_2^2$ and shows that G is firmly nonexpansive.

Second, let G be firmly nonexpansive, then

$$\begin{aligned} \|2G(u) - 2G(v) - (u-v)\|^2 &= 4\|G(u) - G(v)\|^2 - 4\langle G(u) - G(v), u - v\rangle + \|u-v\|^2 \\ &\leq \|u-v\|^2, \end{aligned}$$

which shows that $R := 2G - I$ is nonexpansive, i.e. G is averaged with $\alpha = 1/2$.