Chapter 0
Organization and Introduction

Numerical Methods for Visual Computing
WS 19/20

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Organizational Stuff

Numercial methods for Visual Computing?

Numerical Methods

- Many problems are too large or too complicated to be solved with pen and paper or do not have a closed form solution
- Goal of this lecture: Study different basic types of problems and learn about ways to solve them with a computer
- Understand why such methods work
- Learn how to implement them in Matlab

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Visual Computing

- Synthesizing, reconstructing or analyzing image and video data
- Key technology in many areas
- I'll try to tailor the examples to visual computing

Formalities

Exercises

- Exercise sheets covering the content of the lecture will be passed out every Tuesday
- Exercises contain theoretical as well as programming problems
- You have one week for the exercise sheets and will turn in your solutions on Tuesday
- You may work on the exercises in groups of two
- The solutions will be discussed in the exercises on Friday
- **Reaching at least** 50% **of the total exercise points is required for being admitted to the final exam**
- If solutions have obviously been copied, both groups will get 0 points

Formalities

Questions within the lecture

The more we discuss in the lecture, the more interesting the course will be! Please don't be shy to say something!

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Examination

- Depending on the number of attendees, the final exam will be either oral or written.
- This lecture is worth 5 credits.

Formalities

Miscellaneous

- My office: H-A 7106
- Vaishnavi's office: H-A 7116
- Office hours: Please write an email.
- Lecture: Starts at quarter past. Short break in between.
- Course website:
  `https://www.vsa.informatik.uni-siegen.de/en/numerical-methods-visual-computing-0`
- To access the course material: username: "student", password "100%brain"

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Short lecture overview!

Error analysis and the condition of a problem

Things that can go wrong,

- Example 1: Does the distributive property hold?

$$(x - y)^2 = x^2 - xy - yx + y^2 \ ?$$

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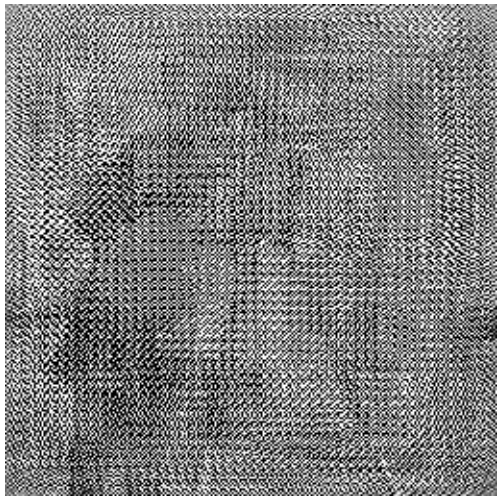

Linear regression

How to fit a linear parametric model to some measured data?

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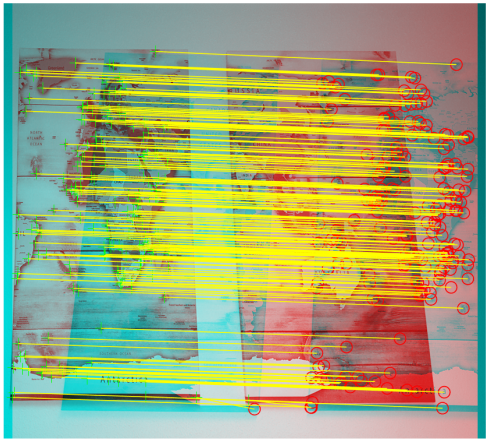


Solve $\min_u \|Au - f\|^2$ and variants thereof

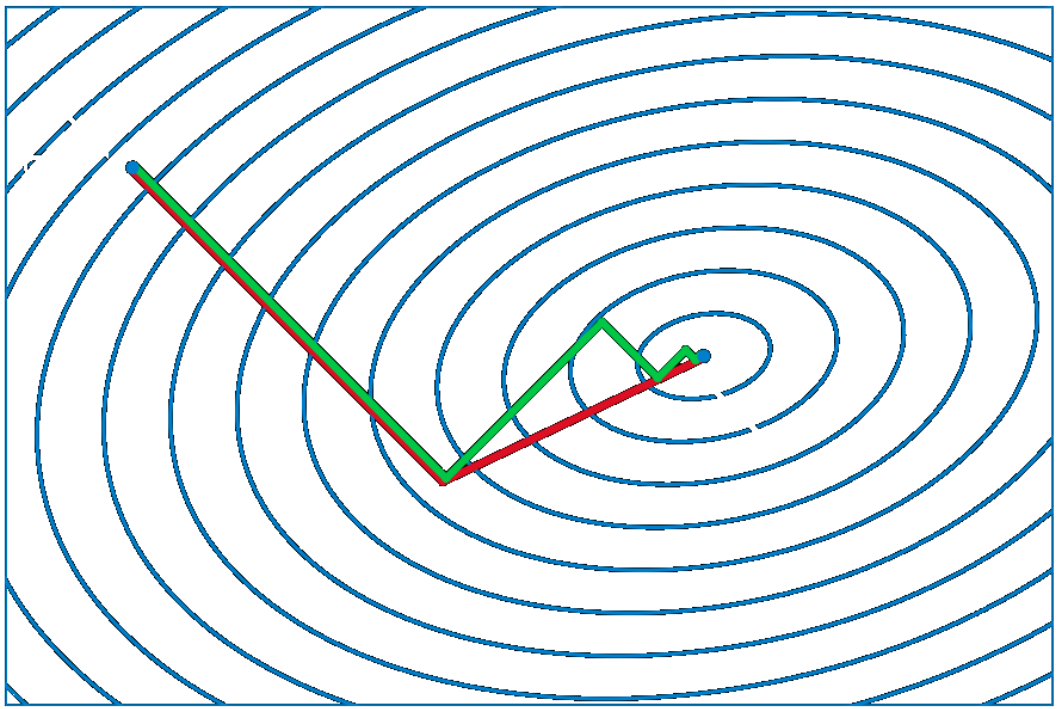
Linear equations

How to solve linear equations

- Exactly, e.g. using Gaussian elimination

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} \tilde{a}_{11} & \tilde{a}_{12} & \tilde{a}_{13} & \tilde{b}_1 \\ 0 & \tilde{a}_{22} & \tilde{a}_{23} & \tilde{b}_2 \\ 0 & 0 & \tilde{a}_{33} & \tilde{b}_3 \end{array}\right]$$

- Iteratively, e.g. using the conjugate gradient method



From Wikipedia: `https://de.wikipedia.org/wiki/CG-Verfahren`

Nonlinear optimization

Assume you have a differentiable but slightly complicated function, e.g.

$$E(\theta) = \sum_{i=1}^{n} (1 - 2y_i) \log\left(1 + \exp(-\langle \theta, x_i \rangle)\right) + \frac{\lambda}{2} \|\theta\|^2$$

and you want to minimize it with respect to θ.

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and you want to minimize it with respect to θ.

As we will see a necessary and sufficient condition is (in this case)

$$\nabla E(\theta) = 0$$

But how can we solve this equation?

Solving nonlinear equations

Our answer here: Determine $\nabla E(u) = 0$ using Newton's method!

Our example: Learn a very simple color skin-detector!

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Computing eigenvalues and eigenvectors

Example: Spectral clustering

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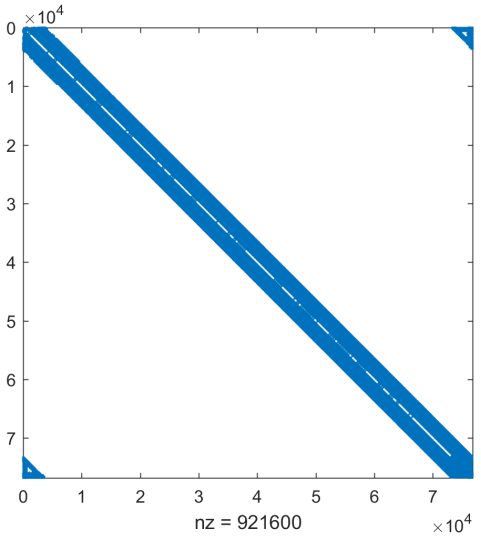

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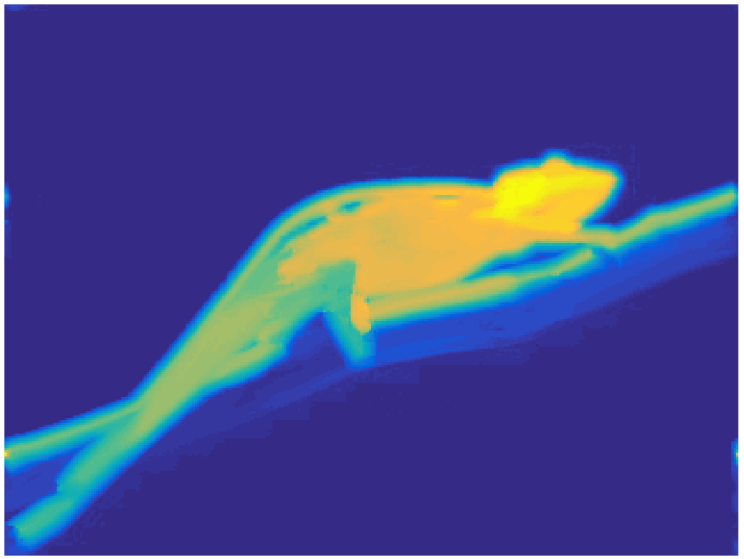
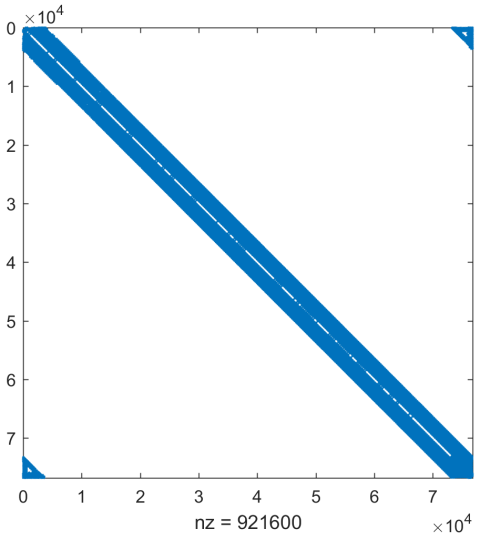
Computing eigenvalues and eigenvectors

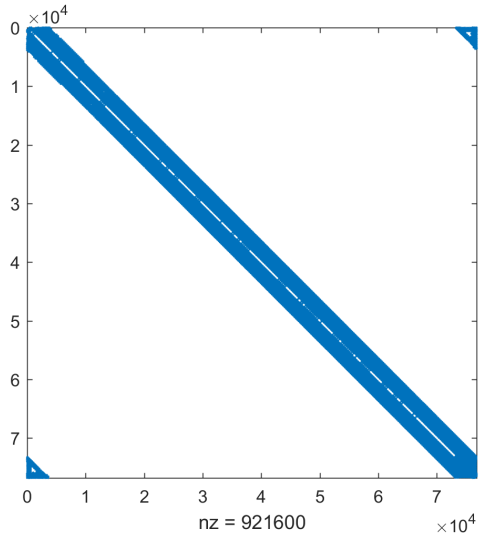
Example: Spectral clustering

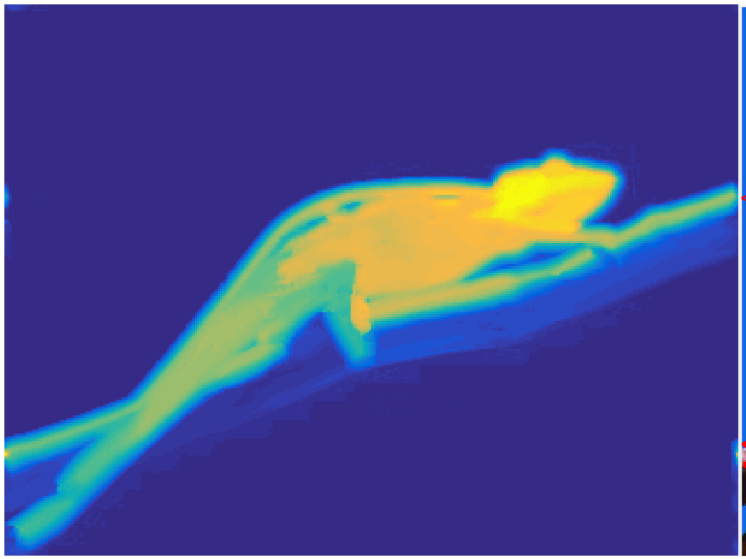


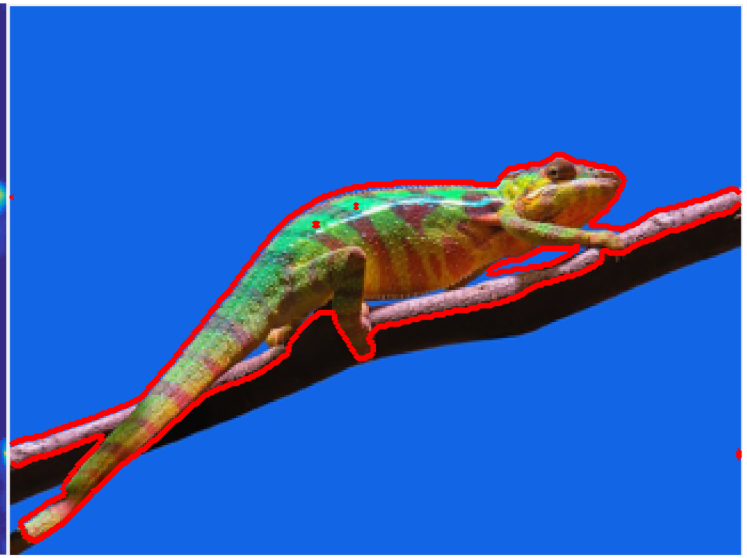


Computing eigenvalues and eigenvectors

Example: Spectral clustering

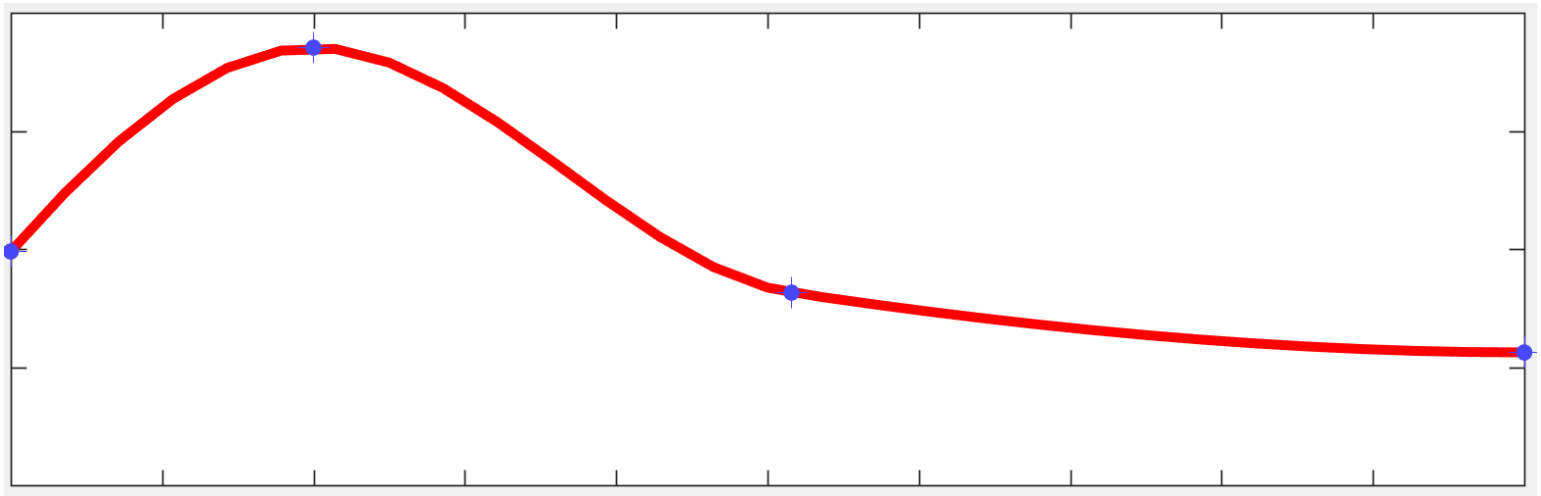






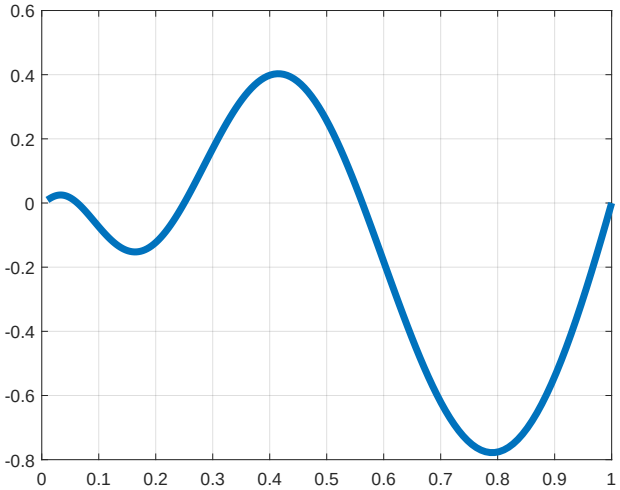
Interpolation

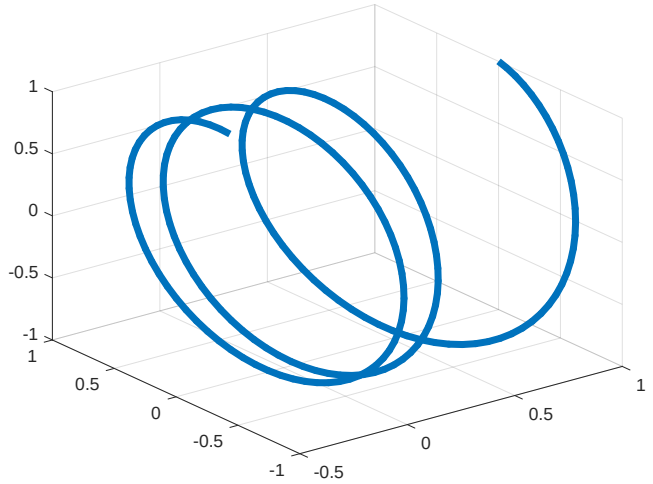
Example: Image filter gui with drag and drop points on a curve



Integration

How to compute the integral over a function numerically?
How to compute the line integral over a function?





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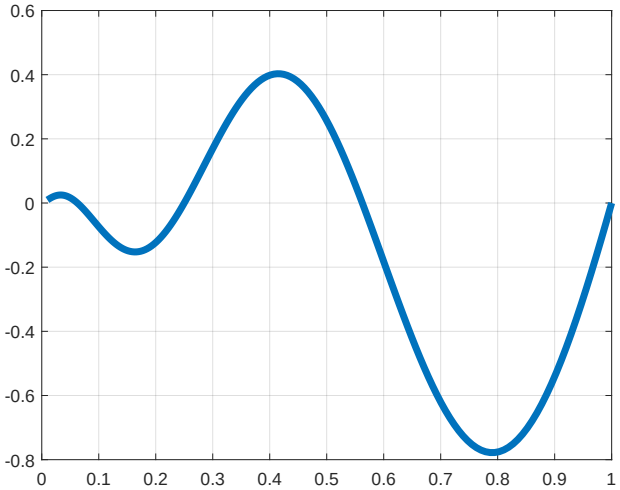

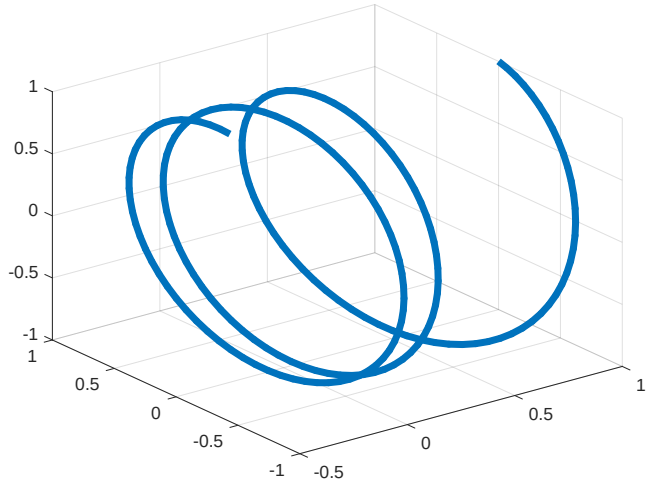

In higher dimensions: What is the volume under a surface?