

Weekly Exercises 10

Room: HA-7116

Wednesday, 05.07.2017, 12:15-14:00,

Submission deadline: Monday, 03.07.2017, 12:15, in the lecture

Theory

Exercise 1 (4 Points). Show that for closed, proper and convex $G : \mathbb{R}^n \rightarrow \mathbb{R} \cup \{\infty\}$, $F^* : \mathbb{R}^m \rightarrow \mathbb{R} \cup \{\infty\}$ the operator

$$T : \mathbb{R}^{m+n} \rightarrow \mathcal{P}(\mathbb{R}^{m+n})$$
$$\begin{pmatrix} u \\ p \end{pmatrix} \mapsto \begin{pmatrix} \partial G & K^T \\ -K & \partial F^* \end{pmatrix} \begin{pmatrix} u \\ p \end{pmatrix} = \begin{pmatrix} \partial G(u) + \{K^T p\} \\ \{-Ku\} + \partial F^*(p) \end{pmatrix}$$

is monotone.

Exercise 2 (2 Points). Let L be an invertible matrix, and let $T : \mathbb{R}^n \rightarrow \mathcal{P}(\mathbb{R}^n)$ be a set-valued monotone operator. Show that

$$L^{-T} T L^{-1} : \mathbb{R}^n \rightarrow \mathcal{P}(\mathbb{R}^n)$$
$$z \mapsto \{L^{-T} p \in \mathbb{R}^n \mid p \in T(L^{-1} z)\}$$

is also monotone.

Exercise 3 (4 Points). Prove that – under our usual assumptions – the algorithm

$$\begin{aligned} u^{k+1} &= \text{prox}_{\tau G}(u^k - \tau K^* \bar{p}^k), \\ p^{k+1} &= \text{prox}_{\sigma F^*}(p^k + \sigma K u^{k+1}), \\ \bar{p}^{k+1} &= 2p^{k+1} - p^k. \end{aligned} \tag{PDHG*}$$

converges, and the limit of the u^k is a minimizer of $G(u) + F(Ku)$.

Hint: Show that (PDHG) is an algorithm we discussed in the lecture applied to a reformulated problem! You do not need to repeat/redo the results from the lecture.*

Programming: TV- ℓ^1 denoising

Exercise 4 (6 Points). Implement a solution of

$$\min_u \|u - f\|_1 + \alpha \|Du\|_1$$

where D is a finite difference approximation of the gradient of an image.

To test you implementation load a clean image, add salt-and-pepper noise by using Matlabs `imnoise(I, 'salt & pepper')`, and try to remove the noise again.