Numerical Methods for Visual Computing
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Winter Semester 17/18

Weekly Exercises 10

To be discussed on Friday, 19.01.2018, 10:15-11:45, in room H-C 6336
Submission deadline: Tuesday, 16.012018, in the lecture

Theory

Exercise 1 (4 points)**.** Let V be a vector space with inner product $\langle\cdot,\cdot\rangle$ and induced norm $\|\cdot\|$. Let $v_1,\dots,v_n$ be vectors with

$$\langle v_i, v_j\rangle = 0, \quad \forall i \neq j, \qquad \|v_i\| = \sqrt{\langle v_i, v_i\rangle} = 1, \quad \forall i.$$

Let $f \in V$ be arbitrary. Show that

$$p = \sum_i \langle v_i, f\rangle v_i$$

is a best approximation of f in $\text{span}(v_1,\dots,v_n)$.

Hints: First show that $\langle f-p, v\rangle = 0$ *for all* $v \in span(v_1,\dots,v_n)$*. Then expand* $\|f-v\|^2 = \|(f-p)-(v-p)\|^2$.

Programming

Exercise 2 (4 points)**.** Approximate the function

$$f(x) = \text{sign}(x)\ \sin(\pi\sqrt{|x|})$$

on the interval $[-1,1]$ by

- Uniformly sampling the function f at n points and computing an interpolation polynomial of degree $n-1$.
- Sampling the function f according to the Chebyshev roots and computing an interpolation polynomial of degree $n-1$.
- Approximating the best L^2 approximation by doing a least squares fit of a polynomial of degree $n-1$ to 1000 uniformly sampled evaluations of f.

You may use Matlabs *polyfit* function, and choose a value like $n = 8$. Plot your results and analyze the differences of the three approximations.