Convex Optimization for Computer Vision
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Exercises: H. Bauermeister
Summer Semester 2018

Universität Siegen
Department ETI
Visual Scene Analysis

Weekly Exercises 10

Room: H-F 104/05

Thursday, 21.06.2018, 16:15-17:45,

Submission deadline: Tuesday, 26.06.2018, 18:00

Exercise 1 (4 Points)**.** For the optimization challenge we introduced the huber penalty $h_\varepsilon \colon \mathbb{R} \to \mathbb{R}$ for $\varepsilon > 0$ defined as

$$h_\varepsilon(x) \coloneqq \begin{cases} \frac{x^2}{2\varepsilon} & \text{for } |x| \leq \varepsilon, \\ |x| - \frac{\varepsilon}{2} & \text{otherwise.} \end{cases}$$

Calculate the proximal operator of h_ε, i.e. find an explicit formulation for $\operatorname{prox}_{\tau h_\varepsilon}$, where $\tau > 0$.

Exercise 2 (4 Points)**.** Let $E \colon \mathbb{R}^n \to \mathbb{R} \cup \{\infty\}$ be proper, closed and convex, such that the proximal operator prox_E is well-defined. Show that Moreau's identity holds, i.e.

$$v = \operatorname{prox}_E(v) + \operatorname{prox}_{E^*}(v)\,.$$

This also implies that the convex conjugate E^* of a simple function E is simple as well.

Hint: Formulate the optimality condition for the proximal operator, use the equivalence of $p \in \partial E(u)$ and $u \in \partial E^*(p)$, and interpret p as the solution of a different optimization problem.

Exercise 3 (4 Points)**.** Recall the energy function for the optimization challenge

$$E(u) \coloneqq \frac{\lambda}{2}\|m \cdot (u - f)\|^2 + \sum_{i=1}^{2N} h_\varepsilon((Du)_i) + \beta\|u\|^2.$$

Find the dual formulation of the original minimization problem using the decomposition $E(u) = G(u) + F(Du)$, where we choose

$$G\,(u) \coloneqq \frac{\lambda}{2}\|m \cdot (u - f)\|^2 + \beta\|u\|^2, \qquad F(Du) \coloneqq \sum_{i=1}^{2N} h_\varepsilon((Du)_i)\,.$$

Your result may contain h_ε^* without an explicit formula.

Hint: Notice that G can be regarded as a separable sum w.r.t. the components of u.