Convex Optimization for Computer Vision
Lecture: M. Möller
Exercises: J. Geiping
Summer Semester 2019

Universität Siegen
Department ETI
Computer Vision

Weekly Exercises 10

Room: HA-7116
Thursday, 11.07.2019, 8:30-10:00,
Submission deadline: Wednesday, 10.07.2019, 18:00

Theory

Exercise 1 (4 Points)**.** Show that for closed, proper and convex $G : \mathbb{R}^n \to \mathbb{R} \cup \{\infty\}$, $F^* : \mathbb{R}^m \to \mathbb{R} \cup \{\infty\}$ the operator

$$
\begin{aligned}
T : \mathbb{R}^{m+n} &\to \mathcal{P}(\mathbb{R}^{m+n}) \\
\begin{pmatrix} u \\ p \end{pmatrix} &\mapsto \begin{pmatrix} \partial G & K^T \\ -K & \partial F^* \end{pmatrix} \begin{pmatrix} u \\ p \end{pmatrix} = \begin{pmatrix} \partial G(u) + \{K^T p\} \\ \{-Ku\} + \partial F^*(p) \end{pmatrix}
\end{aligned}
$$

is monotone.

Exercise 2 (2 Points)**.** Let L be an invertible matrix, and let $T : \mathbb{R}^n \to \mathcal{P}(\mathbb{R}^n)$ be a set-valued monotone operator. Show that

$$
\begin{aligned}
L^{-T} T L^{-1} : \mathbb{R}^n &\to \mathcal{P}(\mathbb{R}^n) \\
z &\mapsto \{L^{-T} p \in \mathbb{R}^n \mid p \in T(L^{-1} z)\}
\end{aligned}
$$

is also monotone.

Exercise 3 (4 Points)**.** Prove that – under our usual assumptions – the algorithm

$$
\begin{aligned}
u^{k+1} &= \mathrm{prox}_{\tau G}(u^k - \tau K^* \bar{p}^k), \\
p^{k+1} &= \mathrm{prox}_{\sigma F^*}(p^k + \sigma K u^{k+1}), \\
\bar{p}^{k+1} &= 2p^{k+1} - p^k.
\end{aligned}
\tag{PDHG*}
$$

converges, and the limit of the u^k is a minimizer of $G(u) + F(Ku)$.

Hint: Show that (PDHG*) *is an algorithm we discussed in the lecture applied to a reformulated problem! You do not need to repeat/redo the results from the lecture.*

Programming: TV-ℓ^1 denoising

Exercise 4 (6 Points)**.** Implement a solution of

$$
\min_u \|u - f\|_1 + \alpha \|Du\|_1
$$

where D is a finite difference approximation of the gradient of an image.

To test you implementation load a clean image, add salt-and-pepper noise by using Matlabs `imnoise(I,'salt & pepper')`, and try to remove the noise again.