Numerical Methods for Visual Computing
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Winter Semester 19/20

Weekly Exercises 10

To be discussed on Friday, 20.12.2019, 10:15-11:45, in room H-C 6336
Submission deadline: Tuesday, 17.12.2019, in the lecture

Theory

Exercise 1 (2 points)**.** Compute the gradient and the Hessian of the energy

$$E(a_1, a_2) = \frac{1}{2}\sum_{i=1}^{n}(a_1 \cos(a_2 x_i) - y_i)^2.$$

Programming

In our programming exercise we will test our optimization methods on the task of parameter estimation. We have some model given, here for example, we know that our data behaves like a cosine wave, but we want to compute the parameters of this wave from given measurements. We view this task of finding the 'optimal' parameters as a minimization problem.

Exercise 2 (2 points)**.** You first generate measurements $y_i = a_1^* \cos(a_2^* x_i)$ for points $x_i \in [-1, 1]$, $i = 1, \ldots, 20$ for the ground truth parameters $a_1^* = 0.5$ and $a_2^* = 2\pi$. Now add some Gaussian noise (with $\sigma = 0.02$) to the measurements y_i. Plot the ground truth cosine wave and the distorted cosine wave.
Next generate a random starting point (a_1, a_2) close to the true values via $a_1 = a_1^* + 0.2n_1$ and $a_2 = a_2^* + 2n_2$ for two random numbers n_1, n_2 from a unit Gaussian distribution. Plot the curve that you would get for these initial parameters.

Exercise 3 (6 points)**.** Now try both methods from the lecture for this problem of finding the optimal parameters a_1, a_2. You already computed the gradient and Hessian of the energy in exercise 1.

- Implement the Newton method to solve the minimization of $E(a_1, a_2)$. How close to the true solution do you have to initialize for the Newton method to converge?
- Implement the gradient descent method to solve the minimization of $E(a_1, a_2)$. Pick a reasonably small step size τ, e.g. $\tau = 0.01$. Does the method always converge? Does it always converge to a good solution?