Variational Methods for Computer Vision
Lecture: M. Möller
Exercises: J. Geiping
Winter Semester 16/17

Visual Scene Analysis
Institute for Computer Science
University of Siegen

Weekly Exercises 11

Room: H-C 7326

Wednesday, 18.01.2017, 14:15-15:45

Submission deadline: Monday, 16.01.2017, 16:00 in the lecture

Programming: email to jonas.geiping@uni-siegen.de

Theory: Segmentation and convex relaxation

Exercise 1 (Penalizing the length of the boundary, 2 points)**.** Consider the characteristic function χ_M of a set M of finite perimeter,

$$\chi_M(x) = \begin{cases} 0 & \text{if } x \notin M, \\ 1 & \text{if } x \in M. \end{cases}$$

Show that the total variation of $u = \mu\ \chi_M$ is equal to $\mu \cdot \text{per}(M)$, where $\text{per}(M)$ denotes the perimeter of M.

Exercise 2 (A-posteriori-optimality for convex relaxations, 2 points)**.** Consider the minimization problems

$$\min_{u:\Omega \to \{0,1\}} \int_\Omega (t - f(x))\ u(x)\ dx + \alpha TV(u), \tag{NC}$$

$$\min_{u:\Omega \to [0,1]} \int_\Omega (t - f(x))\ u(x)\ dx + \alpha TV(u). \tag{C}$$

Show that if a minimizer $\tilde{u}$ of (C) is binary, i.e. $\tilde{u}(x) \in \{0, 1\}$ for all x, then $\tilde{u}$ is a global minimizer of the nonconvex function (NC). Generalize this result to an arbitrary energy $E(u)$ minimized over a set M_1 and minimized over a set M_2 with $M_1 \subset M_2$.

Exercise 3 (Non-uniqueness of minimizes)**.** Give an example showing that we cannot expect minimizers of (C) to be unique.

Programming: Two region segmentation

Exercise 4 (8 Points)**.** Implement the minimization of (C) using the projected gradient descent algorithm from the lecture. Remember that this means only one line of your previous gradient descent implementation has to be changed!

Verify your implementation by loading the coins image, imread('coins.png'), add noise, and try to segment the image into coins and background by finding reasonable values for the 'threshold' t and the regularization parameter α.