Convex Optimization for Computer Vision
Lecture: M. Möller
Exercises: J. Geiping
Summer Semester 2017

Universität Siegen
Department ETI
Visual Scene Analysis

# Weekly Exercises 11

Room: HA-7116

Wednessday, 12.07.2017, 12:15-14:00,

Submission deadline: Monday, 10.07.2017, 12:15, in the lecture

**Exercise 1** (4 points). The Huber penalty $h_\varepsilon : \mathbb{R} \to \mathbb{R}$ is given as

$$h_\varepsilon(x) = \begin{cases} \frac{x^2}{2\varepsilon} & \text{if } |x| \leq \varepsilon, \\ |x| - \frac{\varepsilon}{2} & \text{otherwise.} \end{cases}$$

Show that the huber penalty can be expressed as the so-called *infimal convolution* of the functions $f : \mathbb{R} \to \mathbb{R}$ with $f(x) := \frac{x^2}{2\varepsilon}$ and $g : \mathbb{R} \to \mathbb{R}$ with $g(x) := |x|$:

$$h_\varepsilon(x) = \min_y f(x-y) + g(y)$$

**Exercise 2** (2 points). Let an energy $E$ be the *infimal convolution* of two other energies, i.e.,

$$E(x) = \inf_{x_1, x_2, x = x_1 + x_2} E_1(x_1) + E_2(x_2).$$

Show that

$$E^*(p) = E_1^*(p) + E_2^*(p).$$

**Exercise 3** (4 points). Compute the convex conjugate $F^*$ of the function $F : \mathbb{R}^{2N} \to \mathbb{R}$ defined as

$$F(x) := \sum_{i=1}^{2N} h_\varepsilon(x_i).$$

What is the proximity operator of $F^*$?

# Programming: Huber-TV denoising

**Exercise 4** (6 Points). Implement a solution of

$$\min_u \frac{1}{2}\|u - f\|_2^2 + \alpha \sum_{i=1}^{2N} h_\varepsilon((Du)_i)$$

using the PDHG method, with fixed step sizes $\tau$ and $\sigma$ (PDHG), with adaptive step sizes for one function being strongly convex (PDHG2), and for functions with a strongly convex and an $L$-smooth part (PDHG3). In the above, $D$ is a finite difference approximation of the gradient of an image.