Variational Methods for Computer Vision
Lecture: M. Möller
Exercises: J. Geiping
Winter Semester 17/18

Visual Scene Analysis
Institute for Computer Science
University of Siegen

Weekly Exercises 11

Room: H-C 6336

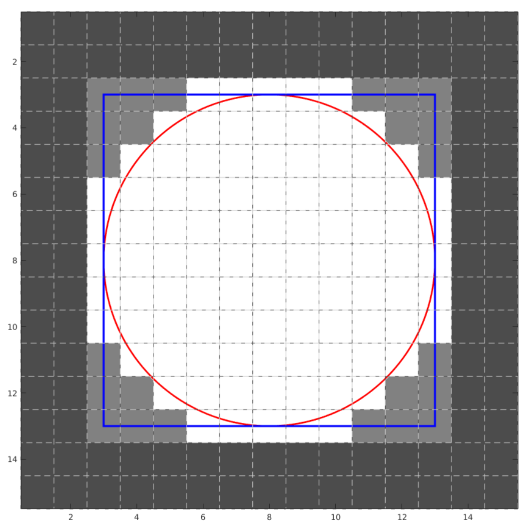
Friday, 19.01.2018, 14:15-15:45

Submission deadline: Tuesday, 16.01.2018, 14:15

Programmming: Email your solution to `jonas.geiping@uni-siegen.de`

Theory: Segmentation and convex relaxation

Exercise 1 (Discrete Perimeter,4 Points)**.** This image contains a grid of 15x15 pixels. Marked are a continuous rectangle, a continuous circle and a discrete rectangle and a discrete circle.



First, compute the perimeter of all four objects: In the lecture we discussed that the total variation of its indicator function is a measure for the perimeter of a set. For our discrete setting, consider the anisotropic total variation of a binary image I:

$$TV_{aniso}(I) = \sum_{i,j}^{m,n} |D_x I| + |D_y I| \tag{1}$$

with the usual definitions for discrete gradients. Use this formula to compute[1] the discrete perimeter of the discrete circle and rectangle.

[1] You may of course first argue why this is equivalent to counting all jumps and then count.

Now argue why the discrete anisotropic perimeter of the circle and square are actually equal. What does this imply for an optimization method that minimizes the perimeter of a set by minimizing the anisotropic total variation?

Exercise 2 (4 points)**.** A-posteriori-optimality for convex relaxations: Consider the minimization problems

$$\min_{u:\Omega\to\{0,1\}} \int_\Omega (t - f(x))\ u(x)\ dx + \alpha TV(u), \tag{NC}$$

$$\min_{u:\Omega\to[0,1]} \int_\Omega (t - f(x))\ u(x)\ dx + \alpha TV(u). \tag{C}$$

Show that if a minimizer $\tilde{u}$ of (C) is binary, i.e. $\tilde{u}(x) \in \{0,1\}$ for all x, then $\tilde{u}$ is a global minimizer of the nonconvex function (NC). Generalize this result to an arbitrary energy $E(u)$ minimized over a set M_1 and minimized over a set M_2 with $M_1 \subset M_2$.

Exercise 3 (2 Points)**.** Give an example showing that we cannot expect minimizers of (C) to be unique.

Programming: Two region segmentation

Exercise 4 (8 Points)**.** Implement the minimization of (C) using the projected gradient descent algorithm from the lecture. Remember that this means only one line of your previous gradient descent implementation has to be changed!

Verify your implementation by loading the coins image, `imread('coins.png')` from the Matlab default images, add noise, and try to segment the image into coins and background by finding reasonable values for the 'threshold' t and the regularization parameter α.