Numerical Methods for Visual Computing
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Winter Semester 18/19

Weekly Exercises 11

To be discussed on Friday, 17.01.2020, 10:15-11:45, in room H-C 6336
Submission deadline: Wednesday, 15.01.2020, in the lecture

Theory

Exercise 1 (4 points)**.** Let V be a vector space with inner product $\langle \cdot, \cdot \rangle$ and induced norm $\| \cdot \|$. Let $v_1, \dots, v_n$ be vectors with

$$\langle v_i, v_j \rangle = 0, \quad \forall i \neq j, \qquad \|v_i\| = \sqrt{\langle v_i, v_i \rangle} = 1, \quad \forall i.$$

Let $f \in V$ be arbitrary. Show that

$$p = \sum_i \langle v_i, f \rangle v_i$$

is a best approximation of f in $\text{span}(v_1, \dots, v_n)$.

Hints: First show that $\langle f - p, v \rangle = 0$ *for all* $v \in span(v_1, \dots, v_n)$. *Then expand* $\|f - v\|^2 = \|(f - p) - (v - p)\|^2$.

Programming

Exercise 2 (6 points)**.** Approximate the function

$$f(x) = \text{sign}(x) \;\; \sin(\pi \sqrt{|x|})$$

on the interval $[-1, 1]$ by

- Uniformly sampling the function f at n points and computing an interpolation polynomial of degree $n - 1$.
- Sampling the function f according to the Chebyshev roots and computing an interpolation polynomial of degree $n - 1$.
- Approximating the best L^2 approximation by doing a least squares fit of a polynomial of degree $n - 1$ to 1000 uniformly sampled evaluations of f.

You may choose $n = 8$ for your experiments. Plot your results and analyze the differences of the three approximations.