Variational Methods for Computer Vision
Lecture: M. Möller
Exercises: J. Geiping
Winter Semester 16/17

Visual Scene Analysis
Institute for Computer Science
University of Siegen

Weekly Exercises 12

Room: H-C 7326

Wednesday, 25.01.2017, 14:15-15:45

Submission deadline: Monday, 23.01.2017, 16:00 in the lecture

Programming: email to jonas.geiping@uni-siegen.de

Theory: Thresholding theorem

The goal of this weeks exercise is to prove the following Chan-Esedoglu-Nikolova thresholding theorem from the lecture:

Consider the minimization problems

$$\min_{u:\Omega\to\{0,1\}} E(u), \tag{NC}$$

$$\min_{u:\Omega\to[0,1]} E(u), \tag{C}$$

for $E(u) = \int_\Omega (t - f(x))\ u(x)\ dx + \alpha TV(u)$. Let $\tilde{u}$ be a minimizer of (C). Then for any $\theta \in]0,1[$ the function $\mathbf{1}_{u>\theta}$ is a global minimizer of (NC)!

Exercise 1 (4 points)**.** Show that the energy E can be rewritten as

$$E(u) = \int_0^1 E(\mathbf{1}_{u>\theta})\ d\theta. \tag{1}$$

Hint: Use the co-area formula and the result from the previous exercise to derive the formula

$$TV(u) = \int_{-\infty}^{\infty} TV(\mathbf{1}_{u>\theta})\ d\theta$$

and use this together with the layer-cake formula.

Exercise 2 (4 points)**.** We proceed to prove the thresholding theorem by contradiction: Let $\tilde{u}$ be a global minimizer of (C). Assume there exists a $\theta_0 \in]0,1[$ such that $E(\chi) < E(\mathbf{1}_{\tilde{u}>\theta_0})$ for χ being a global minimizer of (NC). Then the continuity of $E(\mathbf{1}_{\tilde{u}>\theta})$ with respect to θ implies that there exists an $\epsilon > 0$ such that

$$E(\chi) < E(\mathbf{1}_{\tilde{u}>\theta}) \qquad \forall \theta \in [\theta_0 - \epsilon, \theta_0 + \epsilon]$$

Use formula (1) from exercise 1 to show that the above assumptions imply

$$E(\chi) < E(\tilde{u}).$$

Why is this a contradiction?

Programming: Semi- and fully-automatic segmentation

Exercise 3 (8 Points)**.** The goal of this exercise is to extend your previous implementation of the two region segmentation method in two ways:

1. First, let us consider coding a two region segmentation method, that optimizes for the grey values $c_o \in \mathbb{R}$ and $c_b \in \mathbb{R}$ of the object and background on its own. To do so, consider the function

$$E(u, c_o, c_b) = \sum_i (c_0 - f_i)^2 u_i + (c_b - f_i)^2 (1 - u_i) + \alpha \sqrt{(D_x u)_i^2 + (D_y u)_i^2 + \epsilon^2}$$

   as an energy in u, c_o, and c_b.

   - Derive the optimality conditions for c_o and c_b for a fixed u.
   - Implement an algorithm that alternates between 10 steps of the gradient descent algorithm to find u and updating the constant values c_o and c_b.
   - Update your projected gradient descent to the correct backtracking algorithm as discussed in the lecture.

2. Download the file 'InteractiveSegmentationExample.m' from the course website. Fix the bad idea of computing the histogram of a color image by considering the histograms of the red, green, and blue separately. Are you able to improve the segmentation?