Weekly Exercises 13

Room: H-C 7326

Wednesday, 01.02.2017, 14:15-15:45

Submission deadline: Monday, 30.01.2017, 16:00 in the lecture

Programming: email to jonas.geiping@uni-siegen.de

Theory: Gradient descent and quadratic decoupling

Exercise 1 (4 points)**.** Consider the variational formulation for stereo matching we derived in the lecture

$$\int_\Omega (f_1(x,y) - f_2(x + v(x,y), y))^2 \; dx \; dy + \int_\Omega \sqrt{(\partial_x v(x,y))^2 + (\partial_y v(x,y))^2 + \epsilon^2}$$

with its discrete analogon

$$\sum_{(x,y)\in\tilde{\Omega}} (f_1(x,y) - f_2(x + v(x,y), y))^2 + \sum_{(x,y)\in\tilde{\Omega}} \sqrt{(D_x v(x,y))^2 + (D_y v(x,y))^2 + \epsilon^2}$$

in which f_2 has to remain a function on Ω in order to give sense to all real values $v(x,y)$.

- Derive a formula for the gradient descent iteration for the above minimization problem.
- Your formula will contain a partial derivative of f_2 with respect to the first component. How would you discretize and treat this part numerically?

Exercise 2 (4 points)**.** An alternative to applying gradient descent on the original energy is a strategy called *quadratic decoupling*. One reformulates the energy minimization problem above to

$$\sum_{(x,y)\in\tilde{\Omega}} (f_1(x,y) - f_2(x + v(x,y), y))^2 + \sum_{(x,y)\in\tilde{\Omega}} \sqrt{(D_x u(x,y))^2 + (D_y u(x,y))^2 + \epsilon^2}$$

with the additional constraint $u = v$. If one relaxes this hard constraint to a *soft constraint* by penalizing $\rho\|u - v\|^2$, one obtains an energy in the two variables (u, v) which has an interesting structure: If one fixes the variable u, the minimization problem for v is nonconvex but can be solved pointwise by a brute force search. If one fixes v, the minimization problem for u is a standard smoothed total variation denoising problem.

- Write down the entire energy in u, and v with the soft constraint, along with the two problems that need to be solved, if one applies alternating minimization.

- Assume you are give a function that does an exhaustive search to provide you with a 3D matrix containing the costs

$$C(x, y, v) = (f_1(x, y) - f_2(x + v, y))^2$$

  for all integers $v \in \{0, \cdots, D_{max}\}$. How can you use C to solve your subproblem for v?

Programming: Stereo matching

Exercise 3 (8 Points). Implement the stereo matching strategy you investigated in exercise 2. You can reuse your image denoising code from exercise sheet 3. Example stereo images and code for determining the "brute force" costs C are provided on the course website. $D_{max} = 60$ is a reasonable value for the maximal disparity. For your convenience, the *stereoMatch* functions additionally contains a parameter *windowsize* which allows to determine the costs not on a pixel-by-pixel level, but via patch comparisons.