Variational Methods for Computer Vision
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Winter Semester 16/17

Visual Scene Analysis
Institute for Computer Science
University of Siegen

Weekly Exercises 1

Room: H-C 7326

Wednesday, 26.10.2016, 14:15-15:45

Submission deadline: Monday, 24.10.2016, 16:00 in Room H-C 7327

Theory

Exercise 1 (4 points)**.** Let $A \in \mathbb{R}^{n \times m}$ be a matrix and $f \in \mathbb{R}^n$. Show that $E : \mathbb{R}^n \to \mathbb{R}$, $E(u) = \frac{1}{2}\|Au - f\|^2$ is a convex function.

Exercise 2 (4 points)**.** Let $E : \mathbb{R}^n \to \mathbb{R}$ be a convex function. Show that any local minimizer of E is a global minimizer of E.

Exercise 3 (4 points)**.** Let $A \in \mathbb{R}^{n \times m}$ be a matrix and $f \in \mathbb{R}^n$. Consider the energy

$$E(u) = \frac{1}{2}\|Au - f\|^2,$$

where $\|x\|^2 = \sum_i x_i^2$. Our goal is to determine the optimality condition for minimizinig E.

Let $\hat{u}$ be a minimizer. Naturally, it holds that

$$E(\hat{u}) - E(\hat{u} + \epsilon h) \leq 0$$

for all $\epsilon \in \mathbb{R}$ and $h \in \mathbb{R}^m$.

1. Show that
$$\epsilon\langle A^T(A\hat{u} - f), h\rangle - \frac{\epsilon^2}{2}\|Ah\|^2 \leq 0$$
holds for all $\epsilon \in \mathbb{R}$ and $h \in \mathbb{R}^m$.

2. Divide by ϵ for $\epsilon > 0$ and conclude that
$$\langle A^T(A\hat{u} - f), h\rangle \leq 0.$$

3. Show that
$$A^T(A\hat{u} - f) = 0.$$

Programming

Exercise 4 (4 points)**.** Familiarize yourself with MATLAB.

1. Read the image *ein_ ei.jpg* from the lecture materials into MATLAB and convert it into double format with values in $[0, 1]$.

2. Understand the 'colon' (:) operator and the 'reshape' function. Convert the image I into a vector and back into an image using these functions.

3. Now implement a finite differences Laplace filter Δ. This operator can be written as a filter:

   ```
   Δ = [0 1 0; 1 −4 1; 0 1 0];
   ```

   Compute $\Delta * I$ with

   (a) the 'imfilter' function

   (b) an equivalent matrix-vector multiplication. Write the image as a vector and devise an appropriate matrix.

   Check to make sure both results are equal.

4. Now test your Laplace filter by computing $I - \alpha \Delta I$. What happens for different values of α ?

5. Add noise to the image I by computing

   ```
   I = I + 0.1*rand(size(I));
   ```

   and repeat the experiment in 4. What do you observe ?