Convex Optimization for Computer Vision
Lecture: M. Möller
Exercises: H. Bauermeister
Summer Semester 2018

Universität Siegen
Department ETI
Visual Scene Analysis

Weekly Exercises 1

Room: HF-115

Monday, 16.04.2018, 14:15-15:45,

Submission deadline: Friday, 20.04.2018, letter box at H-A 7116

Theory: Calculus Basics

Exercise 1 (4 Points)**.** Are the following sets open, closed, bounded, compact? Justify your answers!

- $\{(x, y) \in \mathbb{R}^2 \mid x^2 \leq 1,\ y \in]-1, 1[\}$
- $\{v \in \mathbb{R}^n \mid (Av)_i \leq b_i\ \forall i \in \{1, ..., m\}\}$ for some matrix $A \in \mathbb{R}^{m\times n}$, $m < n$, and a vector $b \in \mathbb{R}^m$. (Hint: Careful - you need to distinguish three cases!)

Exercise 2 (2 Points)**.** Let $E : \mathbb{R}^n \to \mathbb{R}$ be differentiable and Lipschitz continuous. Prove that the gradient of E is uniformly bounded, i.e. there exists a constant c such that $\|\nabla E(x)\| \leq c$ for all $x \in \mathbb{R}^n$.

Remark. Note that the reverse is also true: If there exists a constant c such that $\|\nabla E(x)\| \leq c$ for all $x \in \mathbb{R}^n$, then E is Lipschitz continuous!

Exercise 3 (2 Points)**.** Let $f : \mathbb{R}^n \to \mathbb{R}^m$ and $g : \mathbb{R}^l \to \mathbb{R}^n$ be continuous. Prove that $(f \circ g) : \mathbb{R}^l \to \mathbb{R}^m$ is continuous.

Exercise 4 (4 Points)**.** Determine the gradient of the function

$$E(x) = \langle x, Bx\rangle$$

for a matrix $B \in \mathbb{R}^{n\times n}$, by defining

$$E_1 : \mathbb{R}^{2n} \to \mathbb{R}, \qquad E_1(d^1, d^2) = \langle d^1, d^2\rangle$$
$$E_2 : \mathbb{R}^n \to \mathbb{R}^{2n}, \qquad E_2(x) = \begin{pmatrix} x \\ Bx \end{pmatrix},$$

writing $E(x) = E_1(E_2(x))$, and applying the chain rule from the lecture.