

Weekly Exercises 1

Room: HF-115

Thursday, 09.05.2019, 8:30-10:00, H-A 7116,

Submission deadline: Monday, 06.05.2019, 18:00, letter box or office at H-A 7116

Theory: Calculus Basics

Exercise 1 (4 Points). Are the following sets open, closed, bounded, compact? Justify your answers!

- $\{(x, y) \in \mathbb{R}^2 \mid x^2 \leq 1, y \in]-1, 1[\}$
- $\{v \in \mathbb{R}^n \mid (Av)_i \leq b_i \ \forall i \in \{1, \dots, m\}\}$ for some matrix $A \in \mathbb{R}^{m \times n}$, $m < n$, and a vector $b \in \mathbb{R}^m$. (Hint: Careful - you need to distinguish three cases!)

Exercise 2 (2 Points). Let $E : \mathbb{R}^n \rightarrow \mathbb{R}$ be differentiable and Lipschitz continuous. Prove that the gradient of E is uniformly bounded, i.e. there exists a constant c such that $\|\nabla E(x)\| \leq c$ for all $x \in \mathbb{R}^n$.

Remark. Note that the reverse is also true: If there exists a constant c such that $\|\nabla E(x)\| \leq c$ for all $x \in \mathbb{R}^n$, then E is Lipschitz continuous!

Exercise 3 (2 Points). Let $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$ and $g : \mathbb{R}^l \rightarrow \mathbb{R}^n$ be continuous. Prove that $(f \circ g) : \mathbb{R}^l \rightarrow \mathbb{R}^m$ is continuous.

Exercise 4 (4 Points). Determine the gradient of the function

$$E(x) = \langle x, Bx \rangle$$

for a matrix $B \in \mathbb{R}^{n \times n}$, by defining

$$\begin{aligned} E_1 : \mathbb{R}^{2n} &\rightarrow \mathbb{R}, & E_1(d^1, d^2) &= \langle d^1, d^2 \rangle \\ E_2 : \mathbb{R}^n &\rightarrow \mathbb{R}^{2n}, & E_2(x) &= \begin{pmatrix} x \\ Bx \end{pmatrix}, \end{aligned}$$

writing $E(x) = E_1(E_2(x))$, and applying the chain rule from the lecture.