Numerical Methods for Visual Computing
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Weekly Exercises 1

To be discussed on Friday, 19.10.2019, 10:15-11:45, in room H-C 6336/37
Submission deadline: Thursday, 18.10.2019, 10:00, in the 'computer vision' letterbox at H-A 7106

Theory

Exercise 1 (4 points)**.** Consider the function f that maps the parameters (x_1, x_2) to the smaller of the two solutions of the quadratic equation

$$y^2 - 2x_1 y + x_2 = 0$$

under the assumption that $x_1^2 > x_2$.

- Determine an explicit formula for $f(x_1, x_2)$.
- Analyze the stability of the resulting formula by considering

  $$\frac{x_1}{f(x_1,x_2)}\frac{\partial f}{\partial x_1}(x_1,x_2) \qquad \text{and} \qquad \frac{x_2}{f(x_1,x_2)}\frac{\partial f}{\partial x_2}(x_1,x_2).$$

  For which (x_1, x_2) is the problem well-conditioned?
- Guess at which point the following algorithm can run into stability problems even for those (x_1, x_2) at which the problem is not ill-conditioned?

  $$y_1 = (x_1)^2 \tag{1}$$
  $$y_2 = y_1 - x_2 \tag{2}$$
  $$y_3 = \sqrt{y_2} \tag{3}$$
  $$f(x_1, x_2) = x_1 - y_3 \tag{4}$$

- Show that

  $$f(x_1, x_2) = \frac{x_2}{x_1 + \sqrt{x_1^2 - x_2}} \tag{5}$$

  holds.

Programming

Exercise 2 (4 points)**.** Setup your programming environment and implement equations (1)–(4). Then implement an algorithm based on (5). Choose $x_1 = 1$ and $x_2 = 2 \cdot 10^{-8}$, and run both algorithms in double and in single precision. What do you observe?