Numerical Methods for Visual Computing
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Winter Semester 17/18

Weekly Exercises 2

To be discussed on Friday, 03.11.2017, 10:15-11:45, in room H-C 6336
Submission deadline: Monday, 30.10.2017, 18:00, mailbox H-A 7106

Theory

Exercise 1 (4 points). We have seen in the lecture, that minimizing

$$\|Ax - b\|_2^2$$

with respect to x yields the *Gaussian normal equation*

$$A^T Ax = A^T b.$$

In this exercise we will derive the same result from a different perspective. For $A \in \mathbb{R}^{n\times m}$ let $y \in \mathbb{R}^m$ be arbitrary, and consider the function

$$\begin{aligned} E : \mathbb{R} &\to \mathbb{R} \\ \lambda &\mapsto \|A(x + \lambda y) - b\|_2^2 \end{aligned}$$

If $\|Ax - b\|^2$ is minimal, then the function E has to have a minimum at $\lambda = 0$ for any choice of y.

- Remember from school: What is a necessary condition for $E : \mathbb{R} \to \mathbb{R}$ to have a minimum at $\lambda = 0$?
- Show that the necessary condition implies that

  $$\langle y, A^T(Ax - b)\rangle = 0$$

  holds for all $y \in \mathbb{R}^m$.
- Make a specific choice of y to arrive at the Gaussian normal equation.

Exercise 2 (4 points). Solve the following linear equation

$$\begin{pmatrix} 1 & -2 & -3 \\ -1 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ -8 \end{pmatrix}$$

with pen and paper.

Programming

Exercise 3 (4 points)**.** A salesman drives from Zurich to Geneva and records a trip distance of 290km on his tacho; he drives from St. Gallen via Zurich to Geneva and records 370km; he drives from Geneva via Zurich to Chur and records 400km, drives from Chur via Zurich to St. Gallen and records 200km, and drives from Zurich to Chur and records 118km. What is your estimate for the distances from Zurich to Chur, from Zurich to St. Gallen, and from Zurich to Geneva using the information above in a least-squares estimate?

Exercise 4 (4 points)**.** Download the file *exerciseSheet2Code.zip* from the course homepage. You find a .mat file containing two images as well as two lists of points $p^1 \in \mathbb{R}^{2\times 190}$ and $p^2 \in \mathbb{R}^{2\times 190}$. Your goal is to estimate the vector $\vec{a} = (a_1, a_2, \ldots, a_6)$ that determines the best affine transform between p^1, p^2, i.e.

$$\vec{a} = \arg\min_{\vec{a}} \sum_{i=1}^{190} \left\| \begin{pmatrix} a_1 & a_3 \\ a_2 & a_4 \end{pmatrix} \begin{pmatrix} p^1_{1,i} \\ p^1_{2,i} \end{pmatrix} + \begin{pmatrix} a_5 \\ a_6 \end{pmatrix} - \begin{pmatrix} p^2_{1,i} \\ p^2_{2,i} \end{pmatrix} \right\|^2.$$

Hint: First construct a matrix P_i such that

$$P_i\vec{a} = \begin{pmatrix} a_1 & a_3 \\ a_2 & a_4 \end{pmatrix} \begin{pmatrix} p^1_{1,i} \\ p^1_{2,i} \end{pmatrix} + \begin{pmatrix} a_5 \\ a_6 \end{pmatrix}$$

Then stack all P_i to bring the problem into a more standard form.

Once you found the parameter vector $\vec{a}$ you can visualize the image stitching result you obtain with your parameters.