

Weekly Exercises 4

To be discussed on Friday, 09.11.2018, 10:15-11:45, in room H-C 6336
Submission deadline: Tuesday, 06.11.2018, 10:15, H-F 104/105

Theory

Exercise 1 (4 points). In the lecture we discussed that each step of the Gaussian elimination algorithm (without pivoting) can be written as the left multiplication of the current matrix $A^{(j)}$ with a matrix L_j of the form

$$L_j = \begin{pmatrix} 1 & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 1 & 0 & \cdot & \cdot & \cdot & \cdot \\ \cdot & 0 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & -l_{j+1,j} & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & -l_{n,j} & \cdot & 0 & 1 \end{pmatrix} = I - (0, \dots, 0, l_{j+1,j}, \dots, l_{n,j})^T (e_j)^T,$$

where I is the identity and e_j is the j -th unit normal vector. Show that

$$L_j^{-1} = I + (0, \dots, 0, l_{j+1,j}, \dots, l_{n,j})^T (e_j)^T.$$

Assuming your input matrix A allows to carry out the full Gaussian elimination method, conclude that $A = LR$ holds for $L = L_1^{-1} L_2^{-1} \dots L_{n-1}^{-1}$ and R being upper triangular.

Programming

Exercise 2 (4 points + 2 bonus points). Implement the Gaussian elimination algorithm for solving systems of the form $Ax = b$, $A \in \mathbb{R}^{n \times n}$. Check if $\det(A) \neq 0$ at the beginning of your algorithm. If $\det(A) = 0$ output an error message. Within the algorithm itself, if you have to divide by zero to proceed, output an error message.

You can earn 2 bonus points if you additionally implement pivoting.