Numerical Methods for Visual Computing
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Weekly Exercises 4

To be discussed on Friday, 08.11.2019, 10:15-11:45, in room H-C 6336
Submission deadline: Tuesday, 05.11.2019, 10:15, H-F 104/105

Theory

Exercise 1 [4 points]
In the lecture we discussed that each step of the Gaussian elimininination algorithm (without pivoting) can be written as the left multiplication of the current matrix $A^{(j)}$ with a matrix L_j of the form

$$L_j = \begin{pmatrix} 1 & 0 & . & . & . & . & 0 \\ 0 & 1 & 0 & . & . & . & . \\ . & 0 & . & . & . & . & . \\ . & . & . & 1 & . & . & . \\ . & . & . & -l_{j+1,j} & 1 & . & . \\ . & . & . & . & . & . & . \\ 0 & . & . & -l_{n,j} & . & 0 & 1 \end{pmatrix} = I - (0, \dots, 0, l_{j+1,j}, \dots, l_{n,j})^T (e_j)^T,$$

where I is the identity and e_j is the j-th unit normal vector. Show that

$$L_j^{-1} = I + (0, \dots, 0, l_{j+1,j}, \dots, l_{n,j})^T (e_j)^T.$$

Assuming your input matrix A allows to carry out the full Gaussian elimination method, conclude that $A = LR$ holds for $L = L_1^{-1} L_2^{-1} \dots L_{n-1}^{-1}$ and R being upper triangular.

Programming

Exercise 2 [4 points + 2 bonus points]
Implement the Gaussian elimination algorithm for solving systems of the form $Ax = b$, $A \in \mathbb{R}^{n \times n}$. Check if $\det(A) \neq 0$ at the beginning of your algorithm. If $\det(A) = 0$ output an error message. Within the algorithm itself, if you have to divide by zero to proceed, output an error message.

You can earn 2 bonus points if you additionally implement pivoting.