

Weekly Exercises 4

Room: HF-115

Wednesday, 31.05.2017, 12:15-14:00,

Submission deadline: Monday, 29.05.2017, 12:15, in the lecture

Theory: Preparing gradient descent

Exercise 1 (6 points). Let the function $E : \mathbb{R}^n \rightarrow \mathbb{R}$ be given as

$$E(u) := t(u) + h(u).$$

where the function $h : \mathbb{R}^n \rightarrow \mathbb{R}$ is defined as

$$h(u) := g(Du), \quad g(v) = \sum_{i=1}^{2n} \varphi(v_i), \quad \varphi(x) = \sqrt{x^2 + \epsilon^2},$$

with $D \in \mathbb{R}^{2n \times n}$ being a finite difference gradient operator and $t : \mathbb{R}^n \rightarrow \mathbb{R}$ is defined as

$$t(u) := \frac{\lambda}{2} \|u - f\|^2.$$

1. Show that the function E is L -smooth with $L = \lambda + \frac{\|D\|_{S^\infty}^2}{\epsilon}$.
2. Show that the function E is m -strongly convex, with $m = \lambda$.

Exercise 2 (2 Points). Which formula will you implement for the following step in Nesterov's accelerated gradient method: "Find the next $\alpha \in]0, 1[$ by solving

$$\alpha_{k+1}^2 = (1 - \alpha_{k+1})\alpha_k^2 + \frac{m}{L}\alpha_{k+1}"$$

Programming: Image denoising via gradient descent

Exercise 3 (8 Points). Denoise the noisy input image f , given in the file `noisy_input.png` by minimizing the energy from Ex. 3:

$$E(u) = \frac{\lambda}{2} \|u - f\|^2 + \sum_{i=1}^{2n} \sqrt{(Du)_i^2 + \epsilon^2}$$

via

- Gradient descent with a fixed step size $0 < \tau < \frac{2}{m+L}$,
- Gradient descent with backtracking line search
- Nesterovs accelerated gradient method

You can use MATLABs `normest` to estimate the norm $\|D\|_{S^\infty}$ of your finite difference gradient operator D . Plot the decay of energy E against the number of iterations as well as against the time for all three methods. Which one compares favorably?