Numerical Methods for Visual Computing
M. Möller, University of Siegen
Winter Semester 17/18

Weekly Exercises 5

To be discussed on Friday, 24.11.2017, 10:15-11:45, in room H-C 6336
Submission deadline: Tuesday, 21.11.2017, in the lecture

Programming

Exercise 1 (3 points)**.** Implement the Jacobi solver for linear systems of the form

$$Ax = b.$$

You may require A to be symmetric (check this condition in your code!). Make sure to verify that $\|D^{-1}(A-D)\| < 1$ by computing the eigenvalue of largest magnitude.

Generate an exemplary linear system, run your algorithm, and plot the decay of the residual $\|Ax^k - b\|_2$.

Exercise 2 (5 points)**.** Implement the conjugate gradient (CG) method for linear systems of the form

$$Ax = b,$$

with A being symmetric positive definite (which you should check in your code).

Generate an exemplary linear system to which both, CG and the Jacobi method, are applicable. Run both algorithms and compare the decay of the residual $\|Ax^k - b\|_2$, as well as the decay of $\frac{1}{2}\langle x, Ax\rangle - \langle x, b\rangle$.

Exercise 3 (2 bonus points)**.** Generate a linear operator A by setting

```
k = fspecial('gaussian',5,2);
C = @(x)imfilter(x, k, 'circular');
A = @(x)C(C(x));
```

Use the ability of the conjugate gradient method to solve systems matrix free: Generate a blurry image f by applying A to a clean image. Now solve $Ax = C(f)$ as the least-squares solution for determining a sharp image.