

Weekly Exercises 5

To be discussed on Friday, 16.11.2018, 10:15-11:45, in room H-C 6336
Submission deadline: Tuesday, 13.11.2018, 10:15, H-F 104/105

Programming

Exercise 1 (8 points). Implement the Jacobi and Gauß-Seidel solvers for linear systems of the form

$$Ax = b.$$

You may require A to be symmetric (check this condition in your code!). Make sure to verify $\|D^{-1}(A - D)\| < 1$ and $\|L^{-1}(A - L)\| < 1$ by computing the eigenvalue of largest magnitude (using your favorite linear algebra library in your favorite programming language).

Generate an exemplary linear system, run your algorithms, and plot the decay of the residual $\|Ax^k - b\|_2$ over the number of iterations as well as over the runtime of the algorithm. How do the two approaches compare?

Exercise 2 (2 points). Use $\max_i |(Ax^k - b)_i| \leq 10^{-3}$ as a stopping condition for your favorite iterative method from the first exercise. How does it compare to your exact (Gauß-elimination) algorithm from the previous exercise sheet in terms of runtime for matrices $A \in \mathbb{R}^{n \times n}$ for $n \in \{10, 100, 1000\}$?