Convex Optimization for Computer Vision
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Exercises: J. Geiping
Summer Semester 2019

Universität Siegen
Department ETI
Computer Vision

Weekly Exercises 5

Room: H-A 7116

Thursday, 06.06.2019, 8:30-10:00,

Submission deadline: Wednesday, 05.06.2019, 18:00

Theory: Preparing gradient descent

Exercise 1 (6 points)**.** Let the function $E : \mathbb{R}^n \to \mathbb{R}$ be given as

$$E(u) := t(u) + h(u).$$

where the function $h : \mathbb{R}^n \to \mathbb{R}$ is defined as

$$h(u) := g(Du), \qquad g(v) = \sum_{i=1}^{2n} \varphi(v_i), \qquad \varphi(x) = \sqrt{x^2 + \epsilon^2},$$

with $D \in \mathbb{R}^{2n \times n}$ being a finite difference gradient operator and $t : \mathbb{R}^n \to \mathbb{R}$ is defined as

$$t(u) := \frac{\lambda}{2} \|u - f\|^2.$$

1. Show that the function E is L-smooth with $L = \lambda + \frac{\|D\|_{S^\infty}^2}{\epsilon}$.
2. Show that the function E is m-strongly convex, with $m = \lambda$.

Programming: Image denoising via gradient descent

Exercise 2 (8 Points)**.** Denoise the noisy input image f, given in the file `noisy_input.png` by minimizing the energy from Ex. 1:

$$E(u) = \frac{\lambda}{2} \|u - f\|^2 + \sum_{i=1}^{2n} \sqrt{(Du)_i^2 + \epsilon^2}$$

via

- Gradient descent with a fixed step size $0 < \tau < \frac{2}{m+L}$,
- Gradient descent with backtracking line search:
  Proceed as stated in the lecture. Initialize $\tau_k = 1$ and shrink τ_k by the factor β until the condition
  $$E\big(u^k - \tau_k \nabla E\big(u^k\big)\big) \leq E\big(u^k\big) - \alpha \tau_k \|\nabla E\big(u^k\big)\|$$
  is met. Do this for every iterate u^k.

You can use MATLABs `normest` to estimate the norm $\|D\|_{S^\infty}$ of your finite difference gradient operator D. Plot the decay of energy E against the number of iterations as well as against the time for all three methods. Which one compares favorably?