Variational Methods for Computer Vision
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Exercises: J. Geiping
Winter Semester 16/17

Visual Scene Analysis
Institute for Computer Science
University of Siegen

Weekly Exercises 6

Room: H-C 7326

Wednesday, 30.11.2016, 14:15-15:45

Submission deadline: Monday, 28.11.2016, 16:00 in the lecture

Programming: email to jonas.geiping@uni-siegen.de

Theory

Exercise 1 (4 Points)**.** Prove that the conjugate E^* of any energy function $E : \mathbb{R}^n \to \mathbb{R} \cup \{\infty\}$ is convex. Prove that the biconjugate E^{**} of E meets $E^{**}(u) \leq E(u)$ for all u.

Exercise 2 (8 Points)**.** Let $R_1 : \mathbb{R}^n \to \mathbb{R}$ and $R_2 : \mathbb{R}^n \to \mathbb{R}$ be convex and absolutely one-homogeneous, i.e.

$$R_1(\alpha u) = |\alpha| R_1(u) \qquad \forall \alpha,\ u.$$

We will show that the biconjugate of

$$E(u) = \min(R_1(u), R_2(u))$$

is the *infimal convolution*

$$E^{**}(u) = \inf_w R_1(u - w) + R_2(w)$$

between R_1 and R_2. Argue in the following steps:

- The convex conjugate of E is $E^*(p) = \max(R_1^*(p), R_2^*(p))$.
- The convex conjugate of an absolutely one homogeneous function is an indicator function of a convex set, i.e. $R_1^*(p) \in \{0, \infty\}$ for all p. (Hint: First convince yourself that $R_1^*(p) \geq 0$. Then assume there exists a u such that $\langle p, u \rangle - R_1(u) > 0$. Conclude the assertion by considering αu instead).
- Conclude that $\max(R_1^*(p), R_2^*(p)) = R_1^*(p) + R_2^*(p)$.
- Use the Fenchel-Rockafellar duality to show that

  $$\inf_w R_1(u - w) + R_2(w) = \sup_p \langle u, p \rangle - R_1^*(p) - R_2^*(p),$$

  and use the latter to conclude the proof.

Programming

Exercise 3 (4 Points)**.** Implement image demosaicking using infimal convolution regularization as discussed in the lecture, i.e. define

$$R_1(u) = \int_\Omega \sqrt{(0.25 \ \cdot \ \partial_{x_1} u(x))^2 + (\partial_{x_2} u(x))^2} \ dx,$$
$$R_2(u) = \int_\Omega \sqrt{(\partial_{x_1} u(x))^2 + (0.25 \ \cdot \ \partial_{x_2} u(x))^2} \ dx.$$

and solve

$$\min_{u,w} \frac{1}{2}\|Au - f\|_2^2 + R_1(u - w) + R_2(w).$$

using a smoothed version of R_1 and R_2 via your line search gradient descent.