Variational Methods for Computer Vision
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Exercises: J. Geiping
Winter Semester 17/18

Visual Scene Analysis
Institute for Computer Science
University of Siegen

Weekly Exercises 6

Room: H-C 6336

Friday, 01.12.2017, 14:15-15:45

Submission deadline: Tuesday, 28.11.2017, 14:15 in Room H-C 6336

Programmming: Email your solution to `jonas.geiping@uni-siegen.de`

Theory

Exercise 1 (4 Points)**.** Prove that the conjugate E^* of any energy function $E : \mathbb{R}^n \to \mathbb{R} \cup \{\infty\}$ is convex. Prove that the biconjugate E^{**} of E meets $E^{**}(u) \leq E(u)$ for all u.

Programming

This week we will do some repetition of previous exercises.

Exercise 2 (6 Points)**.** Fix your deblurring implementation from last week and extend it to downsampling.

- Generate a downsampling matrix (for example linear downsampling)
- Amend your minimization framework and solve the energy minimization task

$$\min_u ||Ku - f||^2 + \alpha \text{Reg}(u)$$

  with K as your downsampling operator and f noisy downsampled data that you generated, for example from `peppers.png`

- $\text{Reg}(u)$ can be any regularizer! (Re-)implement all regularizers we used so far. These are normal Huber-TV, double-opponent Huber-TV, nonlocal regularization with $L = D - W$ and nonlocal regularization with $L = I - D^{-1/2} W D^{-1/2}$. Solve each optimization by gradient descent.
- Another regularizer is smoothed TV-denoising, defined by

$$\text{Reg}(u) = \sum_{i=1}^{n} \sqrt{(D_x u)_i^2 + (D_y u)_i^2 + \varepsilon^2}$$

  where D_x is the matrix of x-derivatives and D_y the matrix of y derivatives. Think about the correct gradient and implement this regularizer as well.

- Compare PSNR values for these five regularizers for a well chosen α.

Exercise 3 (6 Points)**.** Implement image demosaicking using infimal convolution regularization as discussed in the lecture, i.e. define

$$R_1(u) = \sum_{i=1}^{n} \sqrt{(0.25 \cdot D_x u)_i^2 + (D_y u)_i^2 + \varepsilon^2}$$
$$R_2(u) = \sum_{i=1}^{n} \sqrt{(D_x u)_i^2 + (0.25 \cdot D_y u)_i^2 + \varepsilon^2}$$

and solve

$$\min_{u,w} \frac{1}{2}\|Au - f\|_2^2 + R_1(u - w) + R_2(w).$$

with the usual framework, where A is now the mosaicing operator. Notice how the regularizers are only small modifications of your previous smoothed-TV regularizer from exercise 2.
Remember the hint from the lecture and rewrite your problem with a new variable $v = [u; w]$, so that you can still use your framework.

Hint: A will be a matrix that takes a vectorized color image as input and returns a vectorized gray-scale image with values from the color channels in a Bayer pattern as in the lecture (slide 60).