Numerical Methods for Visual Computing
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Weekly Exercises 6

To be discussed on Friday, 22.11.2019, 10:15-11:45, in room H-C 6336
Submission deadline:Wednesday, 20.11.2019, 10:15

Theory

Exercise 1 (4 points)**.** In preparation of the conjugate gradient method, we will consider a set $\{p_0, \dots, p_{n-1}\} \subset \mathbb{R}^n$ of nonzero vectors that are *mutually conjugate* with respect to a symmetric positive definite matrix A. Your goal is to prove that the p_i are linearly independent. To do so

- Write
$$0 = \sum_{i=0}^{n-1} \alpha_i p_i.$$
What do you need to show about the α_i to conclude the linear independence?
- Use that
$$0 = \langle v, Av \rangle$$
for
$$v = \sum_{i=0}^{n-1} \alpha_i p_i$$
to draw this conclusion.

Programming

Exercise 2 (5 points)**.** Implement the conjugate gradient (CG) method for linear systems of the form
$$Ax = b,$$
with A being symmetric positive definite (which you should check in your code).

Generate an exemplary linear system to which both, CG and the Jacobi method, are applicable. Run both algorithms and compare the decay of the residual $\|Ax^k - b\|_2$, as well as the decay of $\frac{1}{2}\langle x, Ax \rangle - \langle x, b \rangle$.

Exercise 3 (2 bonus points)**.** Generate a linear operator A by setting

```
k = fspecial('gaussian',5,2);
C = @(x)imfilter(x, k, 'circular');
```

```
A = @(x)C(C(x));
```

Use the ability of the conjugate gradient method to solve systems matrix free: Generate a blurry image f by applying C to a clean image. Now solve $Ax = C(f)$ as the least-squares solution for determining a sharp image.

The above code assumes you are programming in matlab. If not, find the corresponding functions for the generation of a 5x5 Gaussian blur kernel k, a function to compute the circular convolution of some image x with k, and define A to be the operator that convolves x with k twice.