Convex Optimization for Computer Vision
Lecture: M. Möller
Exercises: J. Geiping
Summer Semester 2017

Universität Siegen
Department ETI
Visual Scene Analysis

Weekly Exercises 7

Room: HA-7116

Wednessday, 14.06.2017, 12:15-14:00,
Submission deadline: Monday, 12.06.2017, 12:15, in the lecture

Theory

Exercise 1 (4 Points)**.** Looking at commonly used deep learning frameworks like *lasagne*, the Nesterov accelerated gradient method is implemented differently from the version we discussed in the lecture, see `http://lasagne.readthedocs.io/en/latest/modules/updates.html#lasagne.updates.nesterov_momentum`. Find out and describe how the two versions are related.
Hint: Define a variable $z^k = u^{k+1} - u^k$ and identify this variable with the "velocity" in the lasagne formula

Exercise 2 (2 Points)**.** Let $A \in \mathbb{R}^{n\times n}$ be orthonormal, meaning that $A^\top A = AA^\top = I$. Let the convex set C be given as

$$C := \{u \in \mathbb{R}^n : \|Au\|_\infty \leq 1\}.$$

Compute a formula for the projection onto C given as

$$\Pi_C(v) := \text{argmin}_{u\in\mathbb{R}^n} \frac{1}{2}\|u-v\|_2^2, \quad \text{s.t. } u \in C.$$

Exercise 3 (4 Points)**.** Let C_i, $1 \leq i \leq n$ be a family of closed convex sets such that

$$\bigcap_{1\leq i\leq n} C_i \neq \emptyset.$$

Show that the problem of finding an element u^* in the intersection

$$u^* \in \bigcap_{1\leq i\leq n} C_i$$

can be formulated as the following optimization problem:

$$u^* \in \arg\min_{u\in\bigcap_{i\in\mathcal{I}} C_i} \sum_{\substack{j\notin\mathcal{I}\\1\leq j\leq n}} d^2(u, C_j),$$

where $\mathcal{I} \subseteq \{1, 2, \ldots, n\}$ can be arbitrary (including the empty set) and $d(z, X)$ is the distance of a point z to the closed convex set X defined as

$$d(z, X) := \min_{x\in X} \|x - z\|_2.$$

Programming: SUDOKU (6 Points)

Exercise 4 (12 Points)**.** Solve the SUDOKUs given in the files `exampleSudoku1.mat` and `exampleSudoku2.mat` with projected gradient descent. For that you need to find a point

$$u^* \in \bigcap_{1 \le i \le n+m+1} C_i$$

where the convex sets in the intersection are given as

$$C_i := \{u \in \mathbb{R}^{729} : \langle a_i, u \rangle = 1\}, \quad 1 \le i \le n,$$

$$C_i := \{u \in \mathbb{R}^{729} : u_j = 1,\, j \in \mathcal{B}\}, \quad n+1 \le i \le n+m,$$

and $\mathcal{B}$ is the set of indexes corresponding to the known numbers and

$$C_{n+m+1} := \{u \in \mathbb{R}^{729} : u_j \in [0,1],\, \forall\, 1 \le j \le 729\}.$$

For a more precise definition of the constraint sets see lecture.
Solve the programming assignment in the spirit of exercise 3 using the following two partitions of the indexes $\{1, 2, \ldots, n+m+1\}$:

1. $\mathcal{I}_1 := \{n+m+1\}$ and
2. $\mathcal{I}_2 := \{n+1, n+2, \ldots, n+m+1\}$,

and plot the resulting energy decays.
Hint: Show that for the linear constraint sets C_i, $1 \le i \le n$ the distance $d(z, C_i)$ of a point z to the set C_i is equal to $|\langle a_i, z \rangle - 1|$.