

Weekly Exercises 7

Room: H-F 104/05

Thursday, 20.06.2019, 8:30-10:00,

Submission deadline: Wednesday, 19.06.2019, 18:00

Theory

Exercise 1 (4 Points). Looking at commonly used deep learning frameworks like *pytorch*, the Nesterov accelerated gradient method is implemented differently from the version we discussed in the lecture, see https://pytorch.org/docs/stable/_modules/torch/optim/sgd.html. Find out and describe how the two versions are related.

*Hint: Define a variable $z^{k+1} = u^{k+1} - u^k$ and identify this variable with the "velocity" in the *pytorch* formula.*

Exercise 2 (2 Points). Let $A \in \mathbb{R}^{n \times n}$ be orthonormal, meaning that $A^\top A = AA^\top = I$. Let the convex set C be given as

$$C := \{u \in \mathbb{R}^n : \|Au\|_\infty \leq 1\}.$$

Compute a formula for the projection onto C given as

$$\Pi_C(v) := \operatorname{argmin}_{u \in \mathbb{R}^n} \frac{1}{2} \|u - v\|_2^2, \quad \text{s.t. } u \in C.$$

Programming

Exercise 3 (8 Points). Denoise the noisy input image f , given in the file `noisy_input.png` by minimizing the energy

$$E(u) = \lambda \sum_{i=1}^n \sqrt{(u_i - f_i)^2 + \epsilon^2} + \sum_{i=1}^{2n} \sqrt{(Du)_i^2 + \epsilon^2}$$

via

- Gradient descent with a fixed step size,
- Gradient descent with backtracking line search,
- Nesterovs accelerated gradient method.

You can use MATLABs `normest` to estimate the norm $\|D\|_{S^\infty}$ of your finite difference gradient operator D . Plot the decay of energy E against the number of iterations as well as against the time for all three methods. Which one compares favorably?