Weekly Exercises 8

Room: H-C 7326

Wednesday, 14.12.2016, 14:15-15:45

Submission deadline: Monday, 12.12.2016, 16:00 in the lecture

Programming: email to jonas.geiping@uni-siegen.de

Theory

Exercise 1 (4 Points: Derivatives with the Frobinius norm)**.** Consider the energy function

$$E(U) = \|AUB - F\|_F^2$$

where $A \in \mathbb{R}^{n\times m}$, $U \in R^{m\times p}$, $B \in \mathbb{R}^{p\times r}$, and $F \in \mathbb{R}^{n\times r}$ are matrices. The norm $\|\cdot\|_F$ denotes the Frobenius norm, i.e. $\|Z\|_F^2 = \sum_{i,j}(Z_{i,j})^2$.

Determine $\nabla E(U)$ in matrix form.

Possible hint: Remember the Kronecker product:

$$S \otimes T = \begin{pmatrix} S_{1,1}T & \cdots & S_{1,n}T \\ \vdots & \ddots & \vdots \\ S_{m,1}T & \cdot & S_{m,n}T \end{pmatrix}$$

Reduce the problem to a vector problem, derive a rule for $(S \otimes T)^T$, and undo the vectorization.

Exercise 2 (4 Points: Rank-1 Approximation)**.** Consider the problem

$$\hat{U} = \operatorname{argmin}_{U\in\mathbb{R}^{n\times m}} \|U - F\|_F^2 \qquad \text{s.t. } \operatorname{rank}(U) = 1,$$

for $F \in \mathbb{R}^{n\times m}$. Our goal is to derive an explicit formula for $\hat{U}$.

- Show the Frobenius norm is invariant under multiplication with orthonormal matrices.
- Assume that $F = V\Sigma Z^T$ is the (full) singular value decomposition of F. Use the previous property along with a variable substitution to derive an equivalent problem
$$\hat{D} = \operatorname{argmin}_D \|D - \Sigma\|_F \qquad \text{s.t. } \operatorname{rank}(D) = 1.$$
What is the relation between $\hat{D}$ and $\hat{U}$?
- State a formula for $\hat{D}$ and $\hat{U}$[1].

[1]Although it seems quite clear, you may use without a proof that the optimal matrix $\hat{D}$ is diagonal. If you are interested in details consider for instance Mirsky, 1958, "Symmetric Gauge Functions and Unitarily Invariant Norms", Theorem 5.

Live-Programming-Exercise

Please study the implementation of the line search gradient descent algorithm, the derivatives of ℓ^2-squared functions with linear operators, and our two smoothed versions of the ℓ^1-norms with linear operators. Look at the previous programming solution Jonas uploaded and familiarize yourself with MATLAB classes.

Next exercise you will implement an imaging problem with Jonas' help.

This part is not graded. It offers you the possibility to directly address programming as well as derivation questions to Jonas and get your program running at the end of the exercise.