Convex Optimization for Computer Vision
Lecture: M. Möller
Exercises: H. Bauermeister
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Universität Siegen
Department ETI
Visual Scene Analysis

# Weekly Exercises 8

Room: H-F 104/05
Thursday, 07.06.2018, 16:15-17:45,
Submission deadline: Tuesday, 12.06.2018, 18:00

## Theory

**Exercise 1** (4 Points)**.** Let $C_i$, $1 \leq i \leq n$ be a family of closed convex sets such that

$$\bigcap_{1 \leq i \leq n} C_i \neq \emptyset.$$

Show that the problem of finding an element $u^*$ in the intersection

$$u^* \in \bigcap_{1 \leq i \leq n} C_i$$

can be formulated as the following optimization problem:

$$u^* \in \arg \min_{u \in \bigcap_{i \in \mathcal{I}} C_i} \sum_{\substack{j \notin \mathcal{I} \\ 1 \leq j \leq n}} d^2(u, C_j),$$

where $\mathcal{I} \subseteq \{1, 2, \ldots, n\}$ can be arbitrary (including the empty set) and $d(z, X)$ is the distance of a point $z$ to the closed convex set $X$ defined as

$$d(z, X) := \min_{x \in X} \|x - z\|_2.$$

## Programming

**Exercise 2** (8 Points)**.** Solve the SUDOKUs given in the files `exampleSudoku1.mat` and `exampleSudoku2.mat` with projected gradient descent. For that you need to find a point

$$u^* \in \bigcap_{1 \leq i \leq n+m+1} C_i$$

where the convex sets in the intersection are given as

$$C_i := \{u \in \mathbb{R}^{729} : \langle a_i, u \rangle = 1\}, \quad 1 \leq i \leq n,$$

$$C_i := \{u \in \mathbb{R}^{729} : u_j = 1, \, j \in \mathcal{B}\}, \quad n+1 \leq i \leq n+m,$$

and $\mathcal{B}$ is the set of indexes corresponding to the known numbers and

$$C_{n+m+1} := \{u \in \mathbb{R}^{729} : u_j \in [0,1], \forall\, 1 \leq j \leq 729\}.$$

For a more precise definition of the constraint sets see lecture.
Solve the programming assignment in the spirit of exercise 1 using the following two partitions of the indexes $\{1, 2, \dots, n+m+1\}$:

1. $\mathcal{I}_1 := \{n+m+1\}$ and
2. $\mathcal{I}_2 := \{n+1, n+2, \dots, n+m+1\}$,

and plot the resulting energy decays. You may use `sudoku_template.m`.
Hint: Show that for the linear constraint sets $C_i$, $1 \leq i \leq n$ the distance $d(z, C_i)$ of a point $z$ to the set $C_i$ is equal to $|\langle a_i, z\rangle - 1|$.

**Exercise 3** (4 Points)**.** Let $f \in \mathbb{R}^{c \times mn}$ represent the colors of a measured image for each pixel. Suppose that each triple of color values results from a linear combination of specific material signatures stored in a dictionary $A \in \mathbb{R}^{c \times t}$. Each column of $A$ contains the signature of one material. Your task is to find a decomposition $u \in \mathbb{R}^{s \times mn}$ such that $Au$ approximates $f$. Additionally you want $u$ to be sparse, i.e. most of the entries of $u$ should be zero. Therefore implement the following optimization problem

$$\min_u \|Au - f\|^2 + \alpha \|u\|_1,$$

using the provided template in `unmixing_template.m`.