Numerical Methods for Visual Computing
M. Möller, University of Siegen
Winter Semester 18/19

Weekly Exercises 9

To be discussed on Friday, 14.12.2018, 10:15-11:45, in room H-C 6336
Submission deadline: Tuesday, 11.12.2018, in the lecture

Theory

Exercise 1 (4 points)**.** Consider the Newton method for computing the square root of a positive number $a \in \mathbb{R}$, i.e., solving

$$g(u) = u^2 - a = 0$$

using Newton's method. Prove that Newton's method converges for any starting point $u^0 \geq \sqrt{a}$. To do so, consider slide 35 of the lecture and show that the set M of all iterates u^i visited by Newton's method satisfies Banach's fixed point theorem.

Programming

Exercise 2 (4 points)**.** As we will discuss in the lecture, Newton's method can easily be extended to higher dimensions, i.e., $g : \mathbb{R}^n \to \mathbb{R}^n$ by iterating

$$u^{k+1} = u^k - (Jg(u^k))^{-1} g(u^k)$$

where $v = (Jg(u^k))^{-1} g(u^k)$ is the solution to the linear equation

$$(Jg(u^k))v = g(u^k)$$

and

$$Jg(u) = \begin{pmatrix} \frac{\partial g_1}{\partial u_1}(u) & \dots & \frac{\partial g_1}{\partial u_n}(u) \\ \dots & \dots & \dots \\ \dots & \dots & \dots \\ \frac{\partial g_n}{\partial u_1}(u) & \dots & \frac{\partial g_n}{\partial u_n}(u) \end{pmatrix}$$

is the *Jacobian* of g.

Try to solve the equation

$$\begin{pmatrix} x^3 - 3xy^2 - 1 \\ -y^3 + 3x^2 y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

for all starting points $x^0,\ y^0 \in \text{linspace}(-1, 1, 100)$ using Newton's method. Visualize the distance of your solution to each of the true solutions $\hat{u}^1 = (1, 0)^T$, $\hat{u}^2 = (\cos(\frac{4}{3}\pi), \sin(\frac{4}{3}\pi))^T$, and $\hat{u}^3 = (\cos(\frac{4}{3}\pi), -\sin(\frac{4}{3}\pi))^T$.